A GENERALIZED INVERSE STATIC EQUILIBRIUM ANALYSIS FOR PLANAR MECHANISMS

By

RAGHU DWARAKANATH

Bachelor of Engineering

Malaviya Regional Engineering College

University of Rajasthan

Jaipur, Rajasthan, India

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Thesis Approved:

Amoram H. Soni 11/25/87

Thesis Adviser

David G. Lilley

Thomas A. Cook

Lawrence L. Hoberock

Norman N. Durham

Dean of the Graduate College

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NOMENCLATURE

f	displacement constraint
$\dot{f}$	velocity constraint
δf	virtual displacement constraint
F	force
H	horizontal force
$[I]$	identity matrix
$[J]$	jacobian matrix
$[dJ]$	derivative of the jacobian matrix
$[J_o]$	ordinary lagrangian coordinates matrix
$[J_p]$	primary lagrangian coordinates matrix
M	degrees of freedom
M_a	moment about point a
P	power
Φ	lagrangian coordinate
$\dot{\Phi}$	lagrangian velocity
q	generalized coordinate
Q	components of force associated with the generalized coordinate
θ	angle of orientation, if expressed in terms of lagrangian coordinate system it is the lagrangian coordinate
V	vertical force
δW	virtual work

CHAPTER I

INTRODUCTION

A generalized inverse static equilibrium analysis procedure has been developed in this study. The concept of virtual work principle has been conveniently utilized to analyze the equilibrium conditions of planar mechanisms. The use of virtual work principle permits a generalized approach to static equilibrium analysis, which otherwise would be extremely cumbersome.

This study has been conducted to supplement the existing dynamic analysis of planar mechanisms program, Dado (1986) and to present a complete kinematic, dynamic and static analysis package for planar mechanisms. In due course the status of the dynamic analysis program has also been updated as prerequisite to accomodate interactive graphics, user friendliness and the change of the computer system from HP9000 workstation to the Silicon Graphics Iris 3030 workstation.

Thus the package can now perform kinematic, dynamic and static analysis of planar mechanisms, with complete graphics capability and interactiveness. Inverse static equilibrium analysis being the theme of this work, a survey of the past work in this area has been accomplished. The earliest work in this field had been carried out by Livermore (1965). It dealt with the analysis of equilibrium configurations of mechanisms. IMP developed by Sheth and Uicker (1972) deals with the computer aided design of mechanical systems and has the capability of conducting static equilibrium analysis of planar mechanisms. Both the above discussed techniques have been based on the minimum potential energy principle.

Besides these projects, other researchers have worked on similar problems. ADAMS created by Orlandea, Chase and Calahan and the package for kinematic, dynamic and static analysis developed by Amin (1979) deal in detail with the static equilibrium analysis using the vector mechanics approach, which turns out to be very cumbersome to use as far as the convergence of the numerical solution of the problem goes. Study of static force analysis carried out by Razi (1963) using the virtual work principle, as employed here, in this study, is not capable of conducting inverse static equilibrium analysis, but performs the forward static force analysis. In this manuscript, the effort has been made to emphasize the use of virtual work principle in equilibrium analysis.

The chapter that follows discusses the existing features of the the dynamic analysis of planar mechanisms as review and to illustrate the improvements carried out on it. The chapter III deals with the inverse static equilibrium analysis in detail, including the method of generalization. The chapter VI deals with the modeling method used in the program and the input data structure. The chapter V illustrates the applications of the analysis, with examples solved using the same. The chapter VI is the summary and conclusion of this study.

CHAPTER II

A REVIEW OF THE DYNAMIC ANALYSIS OF PLANAR MECHANISMS

This chapter briefly reviews the techniques used by the dynamic analysis of planar mechanisms program and elaborates the new improved structure of the program. The new structure of the program was given shape to accommodate graphics and the inverse static equilibrium analysis. An example in chapter V illustrates the new structure of the program, the complete details of the new structure are available in the Appendix A.

Kinematic Analysis

Kinematic analysis as the title implies emphasizes on position, velocity, and acceleration analysis of the mechanism, it is important because of its application in other sections of the program, besides the vitality of the analysis itself. In this particular analysis the value of the generalized coordinates are determined at an instant of time, with its first and second derivatives, thus arriving at the required value. The position, velocity and acceleration of specified points are also determined using the above generated data. The procedure used for analysis in brief is that the number of constraint equations and the varying coordinates are equal, so the relation between the varying coordinates and the constraints are partially differentiated with respect to the varying coordinates and time, to provide the solution and complete the kinematic analysis. The complex number approach has been used for the kinematic analysis.

Dynamic Analysis

In dynamic analysis the aim is to solve for the kinematic parameters along with the inertial loads, and compute the joint's relations, elemental internal loads and element deflections. The dynamic analysis is of two types forward and inverse. By forward, it means that the mechanism is driven by known motion generators and in case of the inverse dynamics the motion generators are known functions of time or other parameters. In forward dynamic analysis the finite line element technique is used. The mechanism is converted to a line element model and the principles of finite element are used to compute the required results. In case of the inverse dynamics the procedure used here is that of numerical integration of the equations of motion.

Static Analysis

The goal of the static analysis is to determine the final equilibrium position of the mechanism. If the configuration of the mechanism is known and the required parameters are the equilibrium forces the analysis is termed as forward or forward static force analysis and in case of unknown configuration and known forces, it is called inverse analysis or inverse static equilibrium analysis. In this analysis, the forward analysis has been derived from the forward dynamic analysis and employes the same, finite line element technique. The inverse static equilibrium analysis has been incorporated by this study, it employes the virtual work principle and has been discussed in detail in the later chapters of this manuscript. As a final comment on dynamic analysis, it could be said that the new version has not changed the mathematical approach of the original method, but the presentation to the user has been improved and graphics capability has been added.

CHAPTER III

THE INVERSE STATIC EQUILIBRIUM ANALYSIS

The static equilibrium analysis is required in many places as part of the design process of mechanisms. Relevant examples would be a car hood mechanism, which operates in two static equilibrium modes and any mechanical toggle switch mechanism.

To further emphasize the importance of the inverse static equilibrium analysis, consider a robot in any arbitrary position, the action of the gravitational force on the robot's links would tend to collapse the robot. The driving and locking capability provided by the motors at the joints of the mechanism have to counter act the gravity pull to keep the robot in its position, in equilibrium. The inverse static analysis has the capability to analyze such systems to determine the equilibrium position to be reached by the mechanism. The aim of this chapter is to discuss the different techniques available for inverse static equilibrium analysis for planar mechanisms and their applications.

The inverse static equilibrium problems can be attacked in two ways. The first is the vector statics approach and the second method is the analytical statics theory, using the virtual work principle.

Vector Statics

If the resultant force vector and the resultant moment vector about any point applied to a rigid body vanish then the body is in a state of equilibrium. By the elementary laws of mechanics, if a body is initially at rest under the action of a null force system, then it will remain to be in the same state of equilibrium.

In the vector statics approach it is required to consider each rigid body of the system in question separately as a free body and generate the equations of equilibrium for each body. In case of complex systems consisting of many bodies the process turns out to be cumbersome. To exemplify the method consider a four bar linkage as shown in fig. (1), where the horizontal and vertical forces are V and H, acting at the midpoint of links AB and BC. Thus the aim is to find the equilibrium configuration of the mechanism under the influence of the given forces. Mathematically the need is to develop a relationship -

$$F(H, V, \Phi_1, \Phi_2, \Phi_3) = 0 \tag{1}$$

and this being supplemented by loop closure equations -

$$a_1 \sin\theta_1 + a_2 \sin\theta_2 + a_3 \sin\theta_3 = L \tag{2}$$

$$a_1 \cos\theta_1 + a_2 \cos\theta_2 + a_3 \cos\theta_3 = 0 \tag{3}$$

The set of equations are in order to solve for the values of H and V.

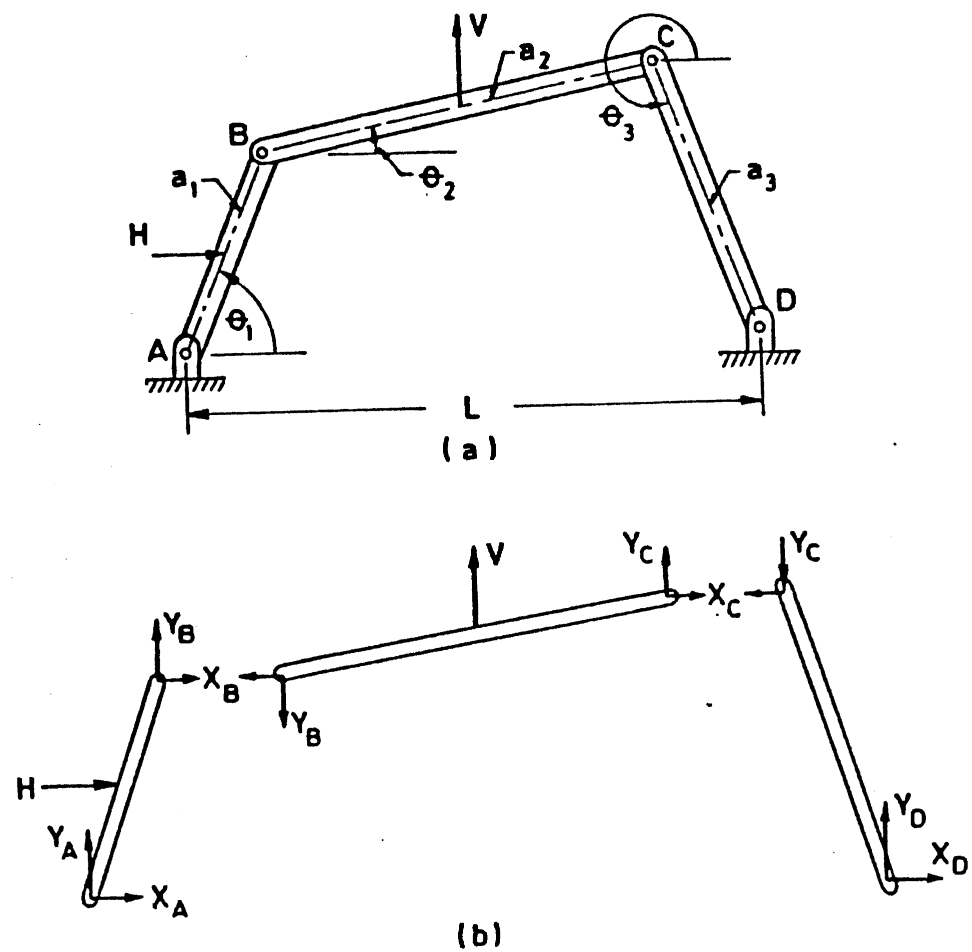

Figure 1. A Four Bar Mechanism in Equilibrium under External Forces

This approach leads to the creation of three equations for each of the free bodies in question. In case of the four bar mechanism this totals up to nine equations, in addition the two loop closure equations, forming a set of eleven equations to be solved.

For the above stated example, the first three of the nine equations can be stated as (the others are identical) -

$$\Sigma X = X_a + H + X_b = 0$$

$$\Sigma Y = Y_a + Y_b = 0$$

$$\Sigma M_a = H\ a_1/2 \sin\theta_1 + X_b\ a_1 \sin\theta_1 + Y_b\ a_1 \cos\theta_1 = 0$$

Thus the eleven equations lead to eleven unknowns namely X_a, Y_a, X_b, Y_b, X_c, Y_c, X_d, Y_d, θ_1, θ_2, and θ_3. These unknowns have to be determined for the values of H and V effective on the system.

This approach, thus seems to be a longer procedure to be used for analysis, though the main advantage of this technique is in the fact that it is easy to generalize, however difficulty arises in the solution of the large set of nonlinear equations it creates.

Analytical Statics

The analytical statics rests on the principle of virtual work. In this study the inverse static equilibrium analysis is based on this technique. This theory has therefore been discussed in great detail to illustrate its use.

If a mechanical system, with lagrangian coordinates

Ξ_i is subjected to a set of kinematic constraints of the form -

$$f_i = (\Xi_1 , \Xi_2 , \Xi_3 , \Xi_4 , . \ . \ . \ . \Xi_M) = 0 \qquad (4)$$

where

i = 1, 2, 3, . . . , number of members

M = degrees of freedom

Then the lagrangian velocities $\dot{\Xi}_j$ must satisfy -

$$\dot{f}_i = \sum_{j=1}^{M} \partial f_i / \partial \Xi_j \ \dot{\Xi}_j = 0 \qquad (5)$$

Any of the $\dot{\Xi}_j$ set, which will satisfy the above equation are termed as virtual velocities and the infinitesimal displacements created by the velocities

$$\delta \Xi_j = \dot{\Xi}_j \ \delta t \qquad (6)$$

in infinitesimal time increment δt are called virtual displacements, which satisfy the equation below.

$$\delta f_i = \sum_{j=1}^{M} \partial f_i / \partial \Xi_j \ \delta \Xi_j = 0 \qquad (7)$$

In a system of M degrees of freedom, the numerical value of M is the number of generalized coordinates and the remaining N lagrangian coordinates are called the secondary coordinates.

no. of lagrangian coordinates = M + N

Thus considering the q_M generalized coordinates and the Ξ_N secondary coordinates, the eqs. (4), (5), and (6) transform to -

$$f_i (\Xi, q) = 0 \qquad (8)$$

$$\sum_{j=1}^{N} \partial f_i/\partial \Xi_j \;\dot{\Xi}_j = -\sum_{j=1}^{M} \partial f_i/\partial q_j \;\dot{q}_j \qquad (9)$$

$$\sum_{j=1}^{N} \partial f_i/\partial \Xi_j \;\delta\Xi_j = -\sum_{j=1}^{M} \partial f_i/\partial q_j \;\delta q_j \qquad (10)$$

The last equation of the above set implies that the secondary virtual displacements $\delta\Xi_j$ may be calculated using the primary or generalized coordinate's virtual displacements, Paul (1979).

The term virtual in virtual displacement or velocities does not necessarily coincide with the actual displacements or velocities, which the coordinate of the system experiences, but are test quantities used to probe the system for results. The following equations expresses the velocity components $(\dot{x}_i, \dot{y}_i)$ of a typical particle P_i as a linear combination of the generalized velocities of the form -

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \sum_{j=1}^{M} \begin{bmatrix} U_{ij} \\ V_{ij} \end{bmatrix} q_j \qquad (11)$$

where U_{ij} and V_{ij} depend on the mechanisms instant configuration.

Redefining the above equation the following conclusion can be arrived at -

$$\begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix} = \sum_{j=1}^{M} \begin{bmatrix} U_{ij} \\ V_{ij} \end{bmatrix} \delta q_j \qquad (12)$$

Therefore any virtual velocity $(\dot{x}_i, \dot{y}_i)$ satisfying the eq. (11) and the virtual displacements $(\delta x_i, \delta y_i)$ satisfying the eq. (12) may deemed to be called virtual velocity or displacement respectively.

With the concept of virtual velocity and virtual displacement, consider a particle with a vector displacement of δr, while a force acts on it, the work done would be -

$$\delta W = F \cdot \delta r \tag{13}$$

where F is the force on the particle during the displacement.

The instantaneous power of the force is given by -

$$P = F . \dot{r} \tag{14}$$

where

$r = \delta r / \delta t$.

If there are N particles in a system and the particle, say j^{th} is located in the base coordinate system (x, y) by the vector $r_j = (x_i, y_i)$ and is acted on by $F_j(x_j, y_j)$, then the power would be stated as -

$$P = \sum_{j=1}^{N} F_j . r_j = \sum_{j=1}^{N} (X_j \dot{x}_j + Y_j \dot{x}_j) \tag{15}$$

For a continuous system of particles comprising of rigid bodies, has to be transformed into an easier form.

Consider a rigid lamina as in fig. (2), where coplanar forces X_j and Y_j act on a typical particle P_j situated at a point (x_j, y_j). The degrees of freedom of this lamina may be described by coordinates x_a and y_a the cartesian coordinates of a point A, fixed on the lamina and its angle of orientation, θ.

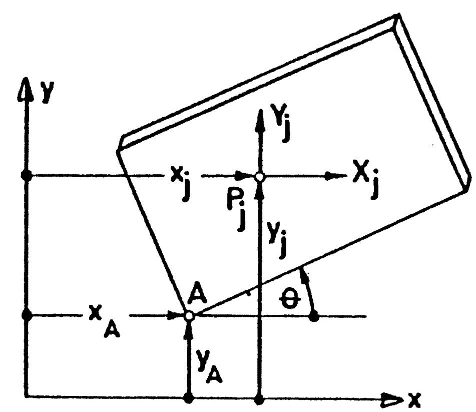

Figure 2. A Rigid Lamina

With any arbitrary motion of the lamina the work done by the forces is at the rate given by -

$$P = \sum_{j=1}^{N} (X_j \dot{x}_j + Y_j \dot{y}_j) \tag{16}$$

where

N is the number of rigid bodies in the system.

but from kinematics it follows that -

$$\dot{x}_j = \dot{x}_a - (y_j - y_a)\,\dot{\theta} \tag{17}$$

$$\dot{y}_j = \dot{y}_a + (x_j - x_a)\,\dot{\theta} \tag{18}$$

which gives the result as -

$$P = X\dot{x}_a + Y\dot{y}_a + M_a\dot{\theta} \tag{19}$$

where

X and Y are

$$\sum_{j=1}^{N} X_j \tag{20}$$

$$\sum_{j=1}^{N} Y_j \tag{21}$$

$$M_a = \sum_{j=1}^{N} (Y_j (X_j - X_a) - X_j (Y_j - Y_a)) \tag{22}$$

and

N is the number of rigid bodies in the system.

The equations stand for the resultant forces and moments of the applied forces about point A. Generalizing the above result of a system of M laminae.

The power will be given by -

$$P = \sum_{i=1}^{M} (X_i \dot{x}_i + Y_i \dot{y}_i + M_i \dot{\theta}_i)$$

For a constrained system with F degrees of freedom, the set of virtual velocities may be expressed in terms of the generalized velocities as in eq. (11), but with the addition of an element for angular virtual velocity θ_i on the left hand side of the equation and Ω_{ij} on the right side of the equation.

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \sum_{j=1}^{F} \begin{bmatrix} U_{ij} \\ V_{ij} \\ \Omega_{ij} \end{bmatrix} \dot{q}_{ij}$$

On substituting the virtual velocities into the eq. (23), a new form of power equation is derived, stated as -

$$P = \sum_{i=1}^{M} \sum_{j=1}^{F} (X_i U_{ij} + Y_i V_{ij} + M_i \Omega_{ij}) \dot{q}_j \qquad (24)$$

rearranging it as -

$$P = \sum_{j=1}^{F} Q_j \dot{q}_j \qquad (25)$$

where

$$Q_j = \sum_{i=1}^{M} (X_i U_{ij} + Y_i V_{ij} + M_i \Omega_{ij}) \qquad (26)$$

are called the components of generalized force associated with the generalized coordinate q_i.

If in eq. (25) both sides are multiplied by a small time increment, δt. The the virtual work may be expressed as -

$$\delta W = P\delta t = \sum_{j=1}^{F} Q_j \delta q_j \tag{27}$$

It is very evident that virtual work is a scalar product of the vector Q and the incremental vector δq, for the generalized coordinates.

In the practical analysis eq. (26) is utilized, this would be evident by the sample problem solved in this chapter and the others by the computer program in chapter IV.

Thus briefly describing the principle of virtual work. As applied to a constrained system of particles in static equilibrium, then the resultant forces R_j acting on any particle must be zero and their work done must be zero, with a virtual displacement of δr_j.

$$\delta W = \sum_{j=1}^{M} R_j \cdot \delta r_j = 0 \tag{28}$$

This is because the work of the constraint forces through virtual displacement compatible with the system constraints is zero.

Considering the applied forces of the system and the work done due to virtual displacement by them would be as follows -

$$\delta W = \sum_{j=1}^{M} F_i \cdot \delta r_i = 0 \tag{29}$$

which forms the basic principle of virtual work.

To illustrate the principle of virtual work the following solved example is cited, Paul (1979). Consider a multiple pendulum, it is an open kinematic chain, with three links, fig. (3). Each of the links are of length a_i and weight Z_i and the center of gravity for each link is located at the center of the link.

A horizontal force X_p acts at the end of the chain, the outer most point of the chain. θ_i is the angle between the link and the vertical axis. The vertical distance between the reference frame and the center of any link is z_i. When the coordinates undergo a displacement of $\delta\theta_i$, the center of gravity point moves vertically by δz_i and the force X_p by horizontally.

Thus by virtual work -

$$\delta W = Z_1 \delta z_1 + Z_2 \delta z_2 + Z_3 \delta z_3 + X_p \delta x_p = 0 \tag{30}$$

is effective, but

$$z_1 = a_1/2 \cos\theta_1 \tag{31}$$

$$z_2 = a_1 \cos\theta_1 + a_2/2 \cos\theta_2 \tag{32}$$

$$z_3 = a_1 \cos\theta_1 + a_2 \cos\theta_2 + a_3/2 \cos\theta_3 \tag{33}$$

$$x_p = a_1 \sin\theta_1 + a_2 \sin\theta_2 + a_3 \sin\theta_3 \tag{34}$$

differentiating the above eq. (31) to (34), the result is as -

$$\delta z_1 = -a_1/2 \sin\theta_1 \ \delta\theta_1 \tag{35}$$

$$\delta z_2 = - a_1 \sin\theta_1 \ \delta\theta_1 - a_2/2 \sin\theta_2 \ \delta\theta_2 \tag{36}$$

$$\delta z_3 = - a_1 \sin\theta_1 \ \delta\theta_1 - a_2 \sin\theta_2 \ \delta\theta_2 - a_3/2 \sin\theta_3 \ \delta\theta_3 \tag{37}$$

$$\delta x_p = a_1 \cos\theta_1 \ \delta\theta_1 + a_2 \cos\theta_2 \ \delta\theta_2 + a_3 \cos\theta_3 \ \delta\theta_3 \tag{38}$$

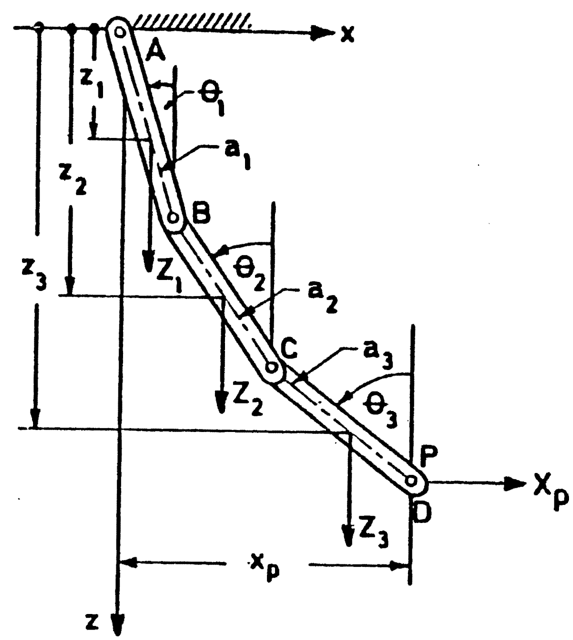

Figure 3. An Open Kinematic Chain

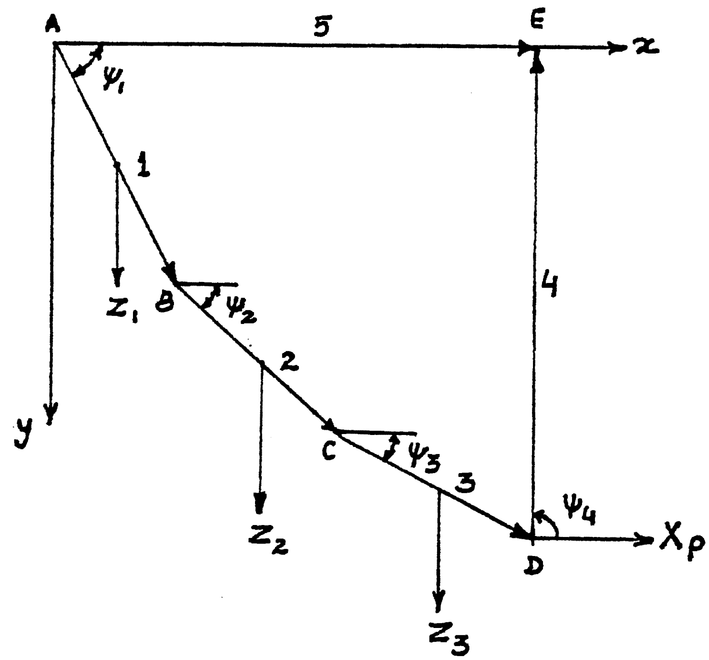

Figure 4. A Kinematic Network Representation

substituting into eq. (30) and simplifying -

$$\delta W = Q_1 \delta\theta_1 + Q_2 \delta\theta_2 + Q_3 \delta\theta_3 \tag{39}$$

is the result, where

$$Q_1 = X_p\ a_1\ \cos\theta_1 - 1/2\ Z_1\ a_1\ \sin\theta_1 - Z_2\ a_1\ \sin\theta_1 - Z_3\ a_1\ \sin\theta_1 \tag{40}$$

$$Q_2 = X_p\ a_2\ \cos\theta_2 - 1/2\ Z_2\ a_2\ \sin\theta_2 - Z_3\ a_2\ \sin\theta_2 \tag{41}$$

$$Q_3 = X_p\ a_3\ \cos\theta_3 - 1/2\ Z_3\ a_3\ \sin\theta_3 \tag{42}$$

The system will be in equilibrium if

$$Q_1 = 0 \quad Q_2 = 0 \quad Q_3 = 0 \tag{43}$$

Eqs. (40) to (43) constitute the solution of the problem. It can be explicitly solved for the coordinates θ_1, θ_2 and θ_3, in terms of the applied forces, as follows -

$$\tan\theta_3 = 2X_p/Z_3 \tag{44}$$

$$\tan\theta_2 = 2X_p/(Z_2+2Z_3) \tag{45}$$

$$\tan\theta_1 = 2X_p/(Z_1+2Z_2+2Z_3) \tag{46}$$

The point to note is that the closed loop problems would have equations transcendental in the displacement variables, thus have to be solved numerically as explained in the following generalization procedure.

Generalization Technique

Consider a mechanism with N degrees of Freedom and M number of rigid links. Then the virtual work principle can be applied as -

$$\sum_{i=1}^{M} F_i \cdot \delta r_i = 0 \tag{47}$$

where

δr_i the virtual displacement

F_i the applied force

The path to the solution would start with the computation of δr_i, the virtual displacements. The procedure for that would be to calculate the jacobian matrix [J], which represents the X and Y coordinate component's summation for each point of action of a force on the rigid member of the system concerned, based on the base reference frame. In mathematical form this may be represented as -

$$[J] = \sum_{i=1}^{M} \sum_{j=1}^{N} (\cos\Phi_i \, L_i + \sin\Phi_i \, L_i) \tag{48}$$

$$j = 1, \ldots, N \text{ dof}$$

where

$L_i \cos\Phi_i$ is the X component of the lagrangian coordinate associated with the link i

$L_i \sin\Phi_i$ is the Y component of the lagrangian coordinate associated with the link i and

L_i is the length of the link i.

Thus mathematically to derive the value of virtual displacement, differential of each element's coordinate location with respect to every lagrangian coordinate of the system, that is -

$$d[J]/d\Phi_k = d/d\Phi_k \left(\sum_{i=1}^{M} \sum_{j=1}^{N} (\cos\Phi_{ij} \, L_i + \sin\Phi_{ij} \, L_i) \right) \tag{49}$$

$$k = 1, \ldots, N \text{ dof}$$

Therefore the derivative jacobian is a set of equations representing the virtual displacement for each lagrangian coordinate and can be expressed as follows -

$$\{\delta r_i\} = [dJ] \{d\Xi_i\} \tag{50}$$

where

[dJ] is a matrix representing the virtual displacement of the system as a set of equations.

$\{\delta r_i\}$ is virtual displacement of the system.

and as stated in eq. (50) it leads to -

$$\{f_i\} \cdot \{\delta r_i\} = 0 \tag{51}$$

The ordinary coordinates, Ξ_o, which are unknown are to be expressed in terms of the primary coordinates, Ξ_p.

Thus based on the virtual work principle the eq. (50) may be turned into -

$$[dJ] \{d\Xi_i\} = 0 \tag{52}$$

The matrix [dJ] now represents the scalar product of the previous [dJ], which represented the virtual displacement and the force vector $\{f_i\}$.

However, at this point it is important to note that the differentiated jacobian matrix or [dJ] formed, contains the number of links as the number of rows and the number of columns are the lagrangian coordinates. The sub-matrices, the ordinary coordinate and primary coordinate matrixes, when separated from this differentiated jacobian matrix, the ordinary coordinate matrix yields a non-square matrix, redundant for the numerical procedures applied later. Thus to rectify this mathematical flaw, the differentiated jacobian matrix is modified to essentially keep the mathematics the same as before, but to reduce its subset, the ordinary coordinate matrix to a square matrix.

Thus rows representing the fixed links and the columns representing the coordinate associated with the those fixed links are eliminated, being fixed they do not have the virtual work principle applicable to them. The mechanism representation process, explained in chapter VI, numbers the fixed links last and same is done with the coordinates associated with it, helping the elimination process. Upon elimination of these rows and columns, the ordinary coordinate matrix formed is a square matrix. Thus enabling the numerical techniques to be applied to it.

Furthermore with the above approach, the eq. (52) may be algebraically simplified to yield -

$$[dJo]\ \{d\Xi_o\} + [dJp]\ \{d\Xi_p\} = 0 \tag{53}$$

future leading to -

$$\{d\Xi_o\} = -\ [dJ_o]^{-1}\ [dJ_p].\{d\Xi_p\} \tag{54}$$

which expresses the ordinary coordinates virtual displacements.

To express the N number of lagrangian coordinates virtual displacements, the above equation is modified to -

$$\begin{matrix} d\Xi_o \\ \cdots\cdots \\ d\Xi_p \end{matrix} = \begin{bmatrix} -\ [dJ_o]\ [dJ_p] \\ [I] \end{bmatrix} \{d\Xi_p\}$$

This gives the final result as far as the virtual displacement vector is concerned.

The prime assumption of the virtual work principle is that the scalar product of the vector of forces and the virtual displacement vector results in a null vector. Using this as the ultimate requirement the complete procedure explained is engulfed in a Newton Raphson numerical technique shell, Paul (1979). To iteratively proceed towards the required solution.

CHAPTER IV

MODELING TECHNIQUES AND INPUT DATA STYLE

It is essential for any generalized mechanism program to establish a consistent modeling technique, which should deal with the topological characteristics of the mechanism. The modeling techniques used in the this analysis conforms to the pattern employed in the dynamic analysis of planar mechanisms. The modeling methods are almost the same as in dynamic analysis to adhere to a consistent method of representation.

Mechanism Representation

The following section details the mechanism representation methods used. It discusses the identification of certain parameters and their use in representing other parameters. Before the identification of these physical or virtual parameters, the definitions of the certain terms is essential.

Definitions

The definitions given below may hold pertinence to inverse static equilibrium analysis and mechanisms only.

<u>Closed loop</u>. In a kinematic chain when a path is traced from one link to another, and it ends at the same starting link. The path traced forms a closed loop.

<u>Element</u>. In inverse static equilibrium analysis, elements are introduced for connectives between joints and are basically subsets of links, they do not perform any functions as they would do in the dynamic analysis.

<u>Joints</u>. The starting or terminal point of a link, element or/and any location on link, where a force/mass acts is termed as a joint. Joints are represented by a circled number.

<u>Lagrangian Coordinates</u>. The lagrangian coordinates are the parameters, which describe the configuration of any mechanism. For any link it is the angular orientation with respect to the frame of reference. For any slider it is the linear component. The coordinates are of two types primary and secondary, and are represented by Φ_p or Φ_o.

<u>Identification of Lagrangian Coordinates</u>

As explained in the definition, the Lagrangian coordinates can be identified. The most important rule to be followed here is that the primary coordinates should be numbered after the ordinary coordinates and the fixed link's coordinates, whether primary or secondary coordinate, should be numbered last. The fig. 21 illustrates the example for inverse static equilibrium analysis's input data style. The coordinates are always represented as Φ_i, where i is the coordinate number.

Identification of Elements

The element is always introduced between two joints, as a subset of a link. It may be introduced for convenience, that is as a virtual element, when a force or a mass acts on a link. The point of action is termed as a joint. The link now would have two parts. It would consist of its initial and terminal joints and the joint signifying the action of a force. These two parts on a link are called elements. In fig. 5, the elements 9, 10, and 11 are introduced to describe the Mechanism's closed loops easily.

Identification of Closed Loops

Mechanisms have to be represented in closed loops due to the technique used for analysis - the complex number approach. If the mechanism is not of a closed loop type, then it has to be filled in by virtual elements to be made a closed loop type of mechanism. The classic example for this case is in fig. 7, the 3-R Robot.

Identification of Joints

The links or elements of the mechanism are initiated or terminated by joints. They are identified by numbers, in a maintained pattern. Joints or virtual ones can be created

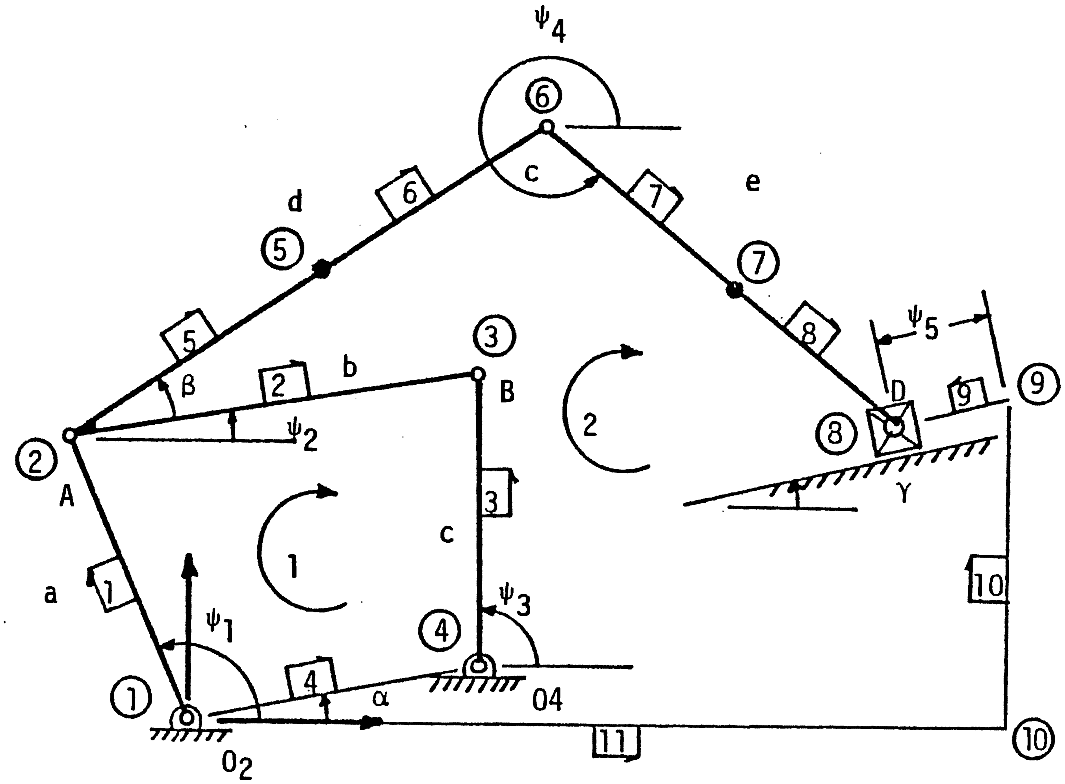

Figure 5. A Description of a Six Bar Mechanism's Model

as in case of the joint 2 in fig. 7 to represent a particular point of interest for analysis, such as where there may be a mass or a force acting.

Modeling of the Slider Path

In mechanisms with slider the path may have to be represented as a moving link. The following procedure is employed to model the same as a line element, consider the straight portions of the moving link between the slider location and any other kinematic pairs on the path as separate links, variable both in length and orientation. The masses should be lumped at the proper locations on these links of variable lengths. Also, consider the length of each element on these links as variable coordinates. The fig. 6, shows moving slider pair modeled by four elements, each being represented by a varying coordinate, they may be related by -

$$\Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 = L_2$$

$$\Phi_3 = \Phi_4$$

and

$$\Phi_5 = \Phi_6$$

The slider modeling technique used above is the same as described by Dado (1986).

Input Data Style

Once the user has developed a complete model of the

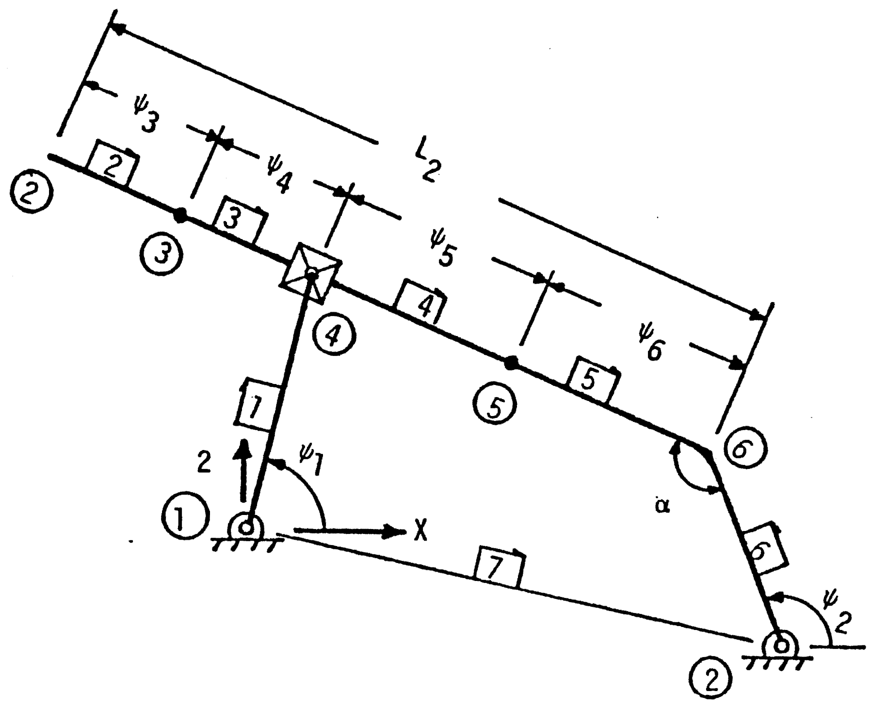

Figure 6. A Mechanism to Illustrate the Variable Length Elements

mechanism to be analyszed, this includes a complete drawing of the same, in all respects. The program's interactive input can be used to enter the data of the mechanism in the standardized form used by the program's number crunching routines. The following menus are presented to the user for the inverse static equilibrium analysis. They briefly explain the input data style.

(1) The characteristic data menu.

(2) The link data menu.

(3) The element data menu.

(4) The loop data menu.

(5) The joint data menu.

(6) The coordinates relation data menu.

(7) The coordinates estimation menu. With reference to the fig. 22, the above menus are explained below, via a sample data input. In the characteristic data menu the data input required are -

(1) Number of moving links = 3

(2) Number of fixed links = 1

(3) Number of kinematic pairs - moving = 2

(4) Number of kinematic pairs - fixed = 2

(5) Number of ordinary coordinates = 3

(6) Number of primary coordinates = 1

(7) Number of loops = 1

(8) Number of elements = 7

(9) Number of joints = 7

(10) Gravity = 381.0

The link data menu requires (for each link prompted in sequence in the program, but here only link 2 associated with the ordinary coordinate 1 is cited) -

(1) Number of elements on the link = 2

(2) Element numbers on the link = 3 and 4

The element data menu requires (for each element prompted in sequence in the program, but here only element 4 is cited) -

(1) Element number = 4

(2) Associated langrangian coordinate = 1

(3) Initial joint = 4

(4) Terminal joint = 5

(5) Element's constant length = 3

(6) Element's constant orientation = 0

(7) Coordinate variation data = 2

The loop data menu requires, for each loop -

(1) Loop number = 1

(2) Number of elements in the loop = 7

(3) Element numbers in the loop = 1, 2, 3, 4, 5, 6 and 7

The joint data menu requires for each joint, (here only joint 4 is cited) -

(1) The joint number = 4

(2) Number of elements in the joint's path = 3

(3) Element number in the path of the joint = 1, 2, and 3

(4) Force magnitude at the joint = 10

(5) Force orientation at the joint = 0

The important point to note here, is that the term force, includes body force also.

The coordinates relation data menu requires, for each link, (here only link 2 is cited) -

(1) Number of links preceeding the link = 1

(2) Link numbers preceeding the link = 1

The coordinate estimation menu requires, for each lagrangian coordinate, (here only coordinate 2 is cited) -

(1) Its estimated value = 0

(2) Fixed or moving link = 1

The above menus take in data from the user and represent it in the data structure used by the computer code. The input data style being interactive is easier to use than creating a data set a file editing procedure.

CHAPTER V

ILLUSTRATIVE APPLICATIONS

Various applications have been solved to test and verify the accuracy of this program. The examples envelope a range of mechanism possibilities. The problems solved are easy in terms of verification.

Illustrative Example - Dynamics Analysis

Since the dynamic analysis of planar machinery has been modified and improved on a different computer system and its structure has been changed to great extend from its earlier appearance. The following example has been cited from Dado (1986) and reworked on the improved version of the dynamic analysis program.

The example analyzes a 3-R planar mixed loop robot, shown in the fig. 7. This example has been solved by the forward dynamic analysis. The end effector at the joint 7 must trace the circular path shown in the figure, with the defined displacement function S. The orientation of the element 6 must remain constant at an angle equal to zero. There are three input actuators, one at joint 1, which defines the orientation of element 1 and the other two are

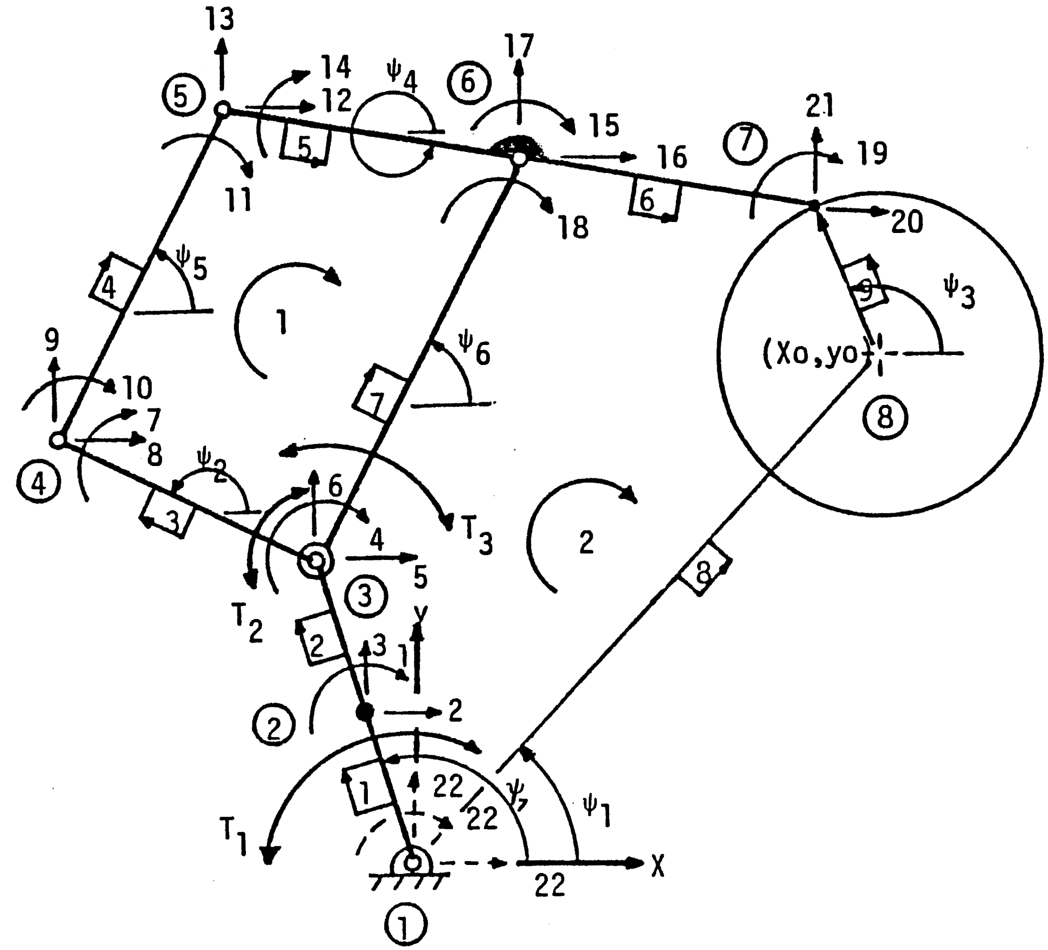

Figure 7. A 3R Mixed Loop Robot

at the double joint 3 and they define the orientations of the elements 3 and 7. The inertias of the elements 1 and 2 are lumped at joint 2 and those for elements 5 and 6 are lumped at joint 6. The inertias of the rest of the elements are neglected. A load of 40 lb-m is lumped at the joint 7 for the end effector assembly. The combined mass and mass moment of inertia of the two actuators at joint 3 are 60 lb-m and 2.8 lb_f-in-sec^2. For the dimensions and the inertias shown in fig. 7, it is necessary to compute the positions, velocities, accelerations, and input torques for the three acuators. The input data for this example has been derived directly from fig. 7.

There are no applied loads or known driving forces or torques. There are two constraints equations and one time-dependent equation for defining the orientation of element 9 using the function S. This constraint equation is as follows -

$$tc(1) = g(\Phi_6, t) = s(t)/r - \Phi_6 = 0$$

The other equation is path constraint and it defines the orientation of the end effector. It is given by -

$$ts(1) = s(\Phi_9) = \Phi_{8o} - \Phi_9 = 0$$

Finally, the derivative of these two constraints equations are

$$ptc(1,6) = \partial g/\partial\Phi_6 = -1$$

$$pttc(1) = \partial g/\partial t = 1/r\ \partial s(t)/\partial t$$

$$dptc(1,6) = d/dt\ \partial g/\partial t = 0$$

$$dpttc(1) = d/dt\ \partial g/\partial t = 1/r\ d/dt\ \partial s(t)/\partial t$$

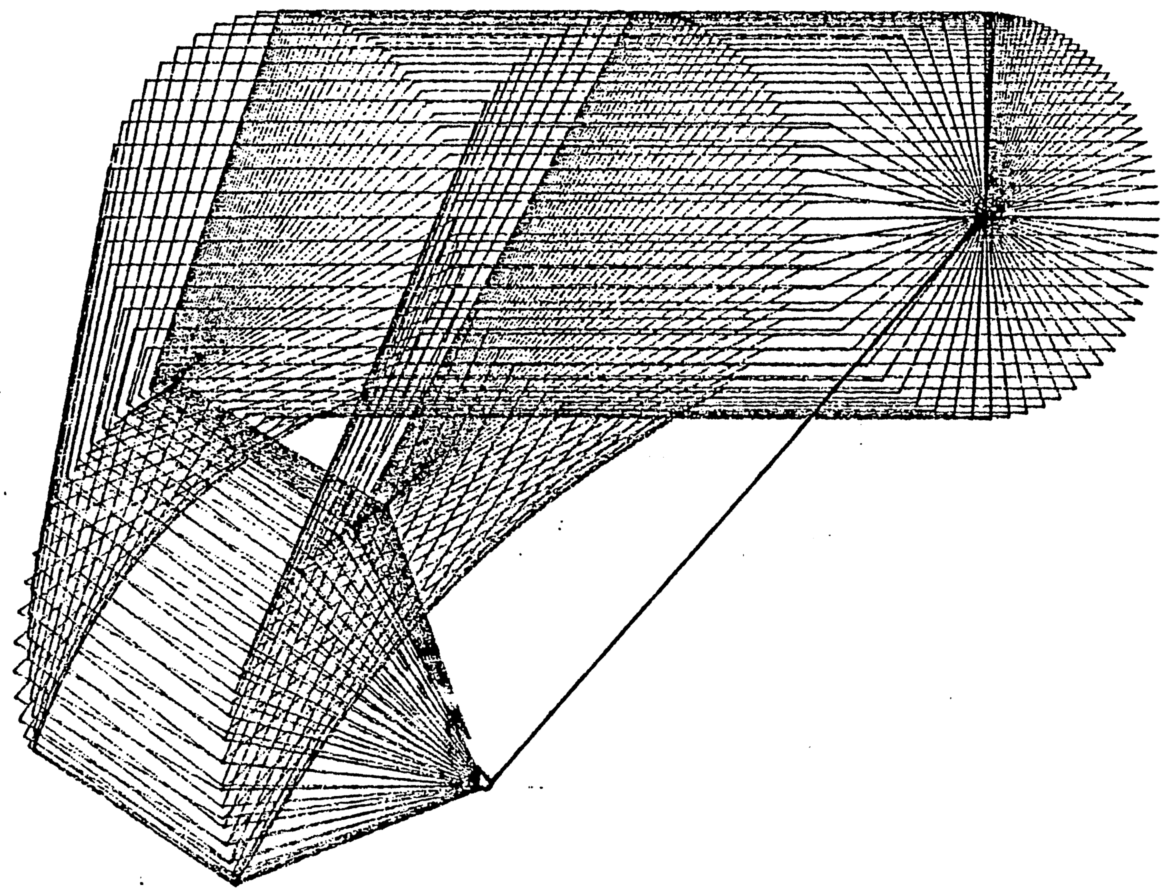

Figure 8.   A Sample Animation of the 3R Mixed Loop Robot

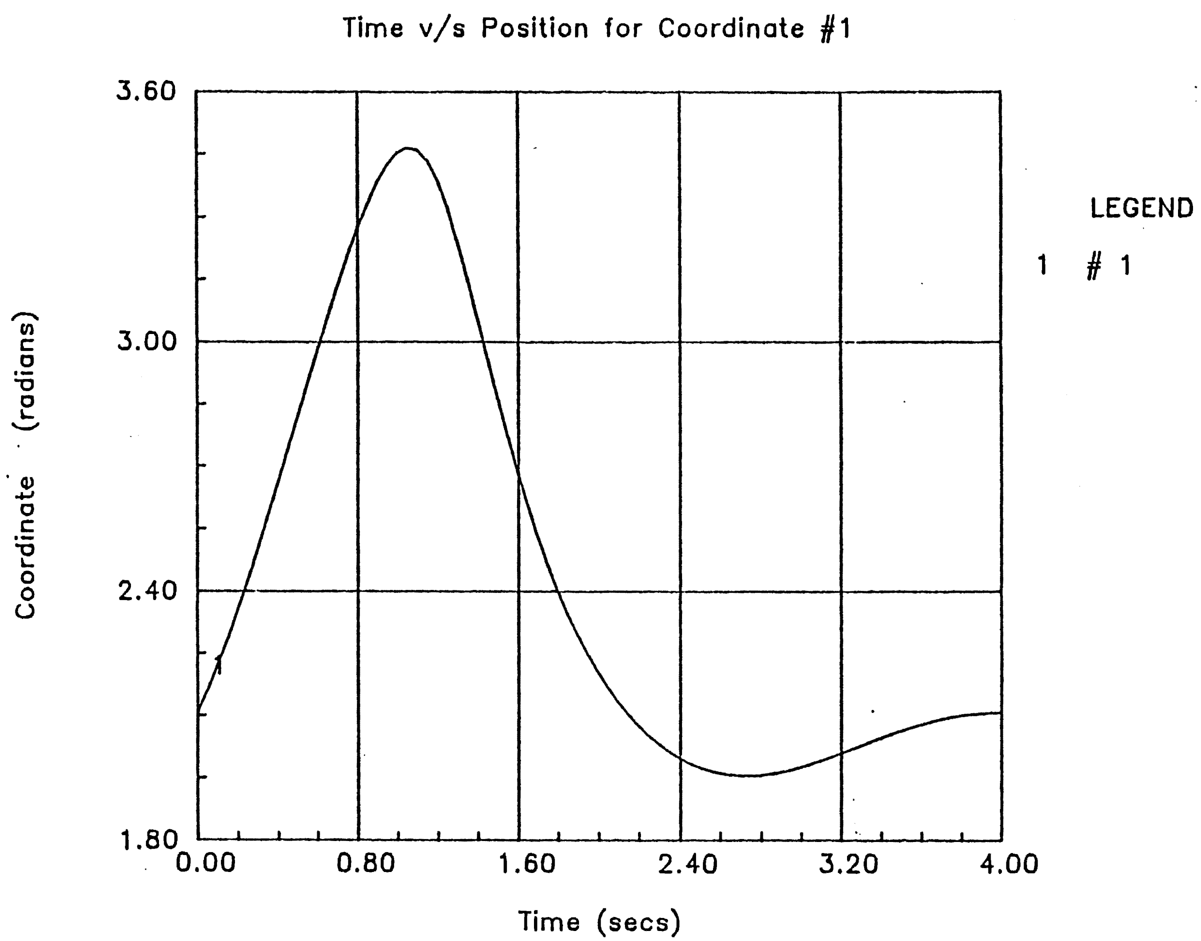

Figure 9. A Plot of Position v/s Time for Coordinate #1

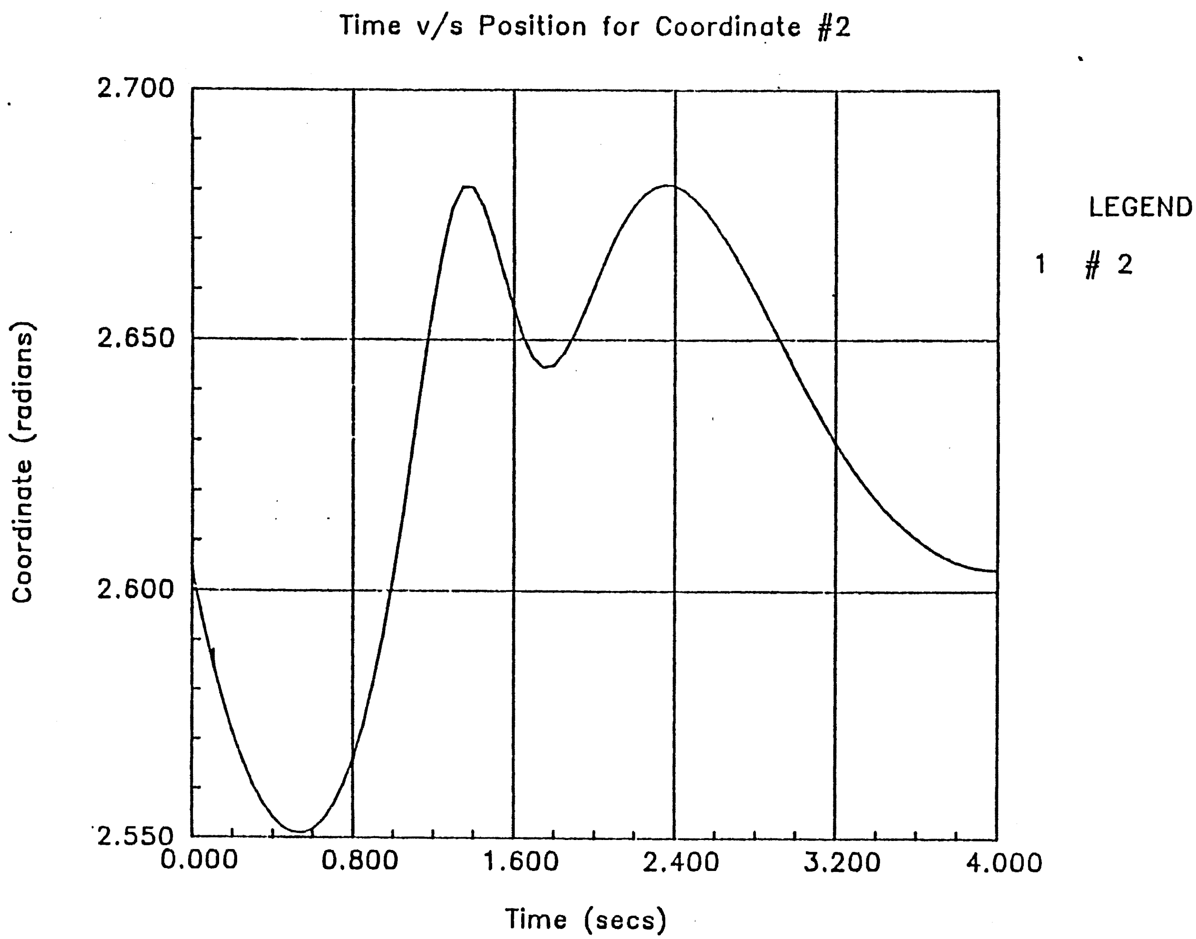

Figure 10. A Plot of Position v/s Time for Coordinate #2

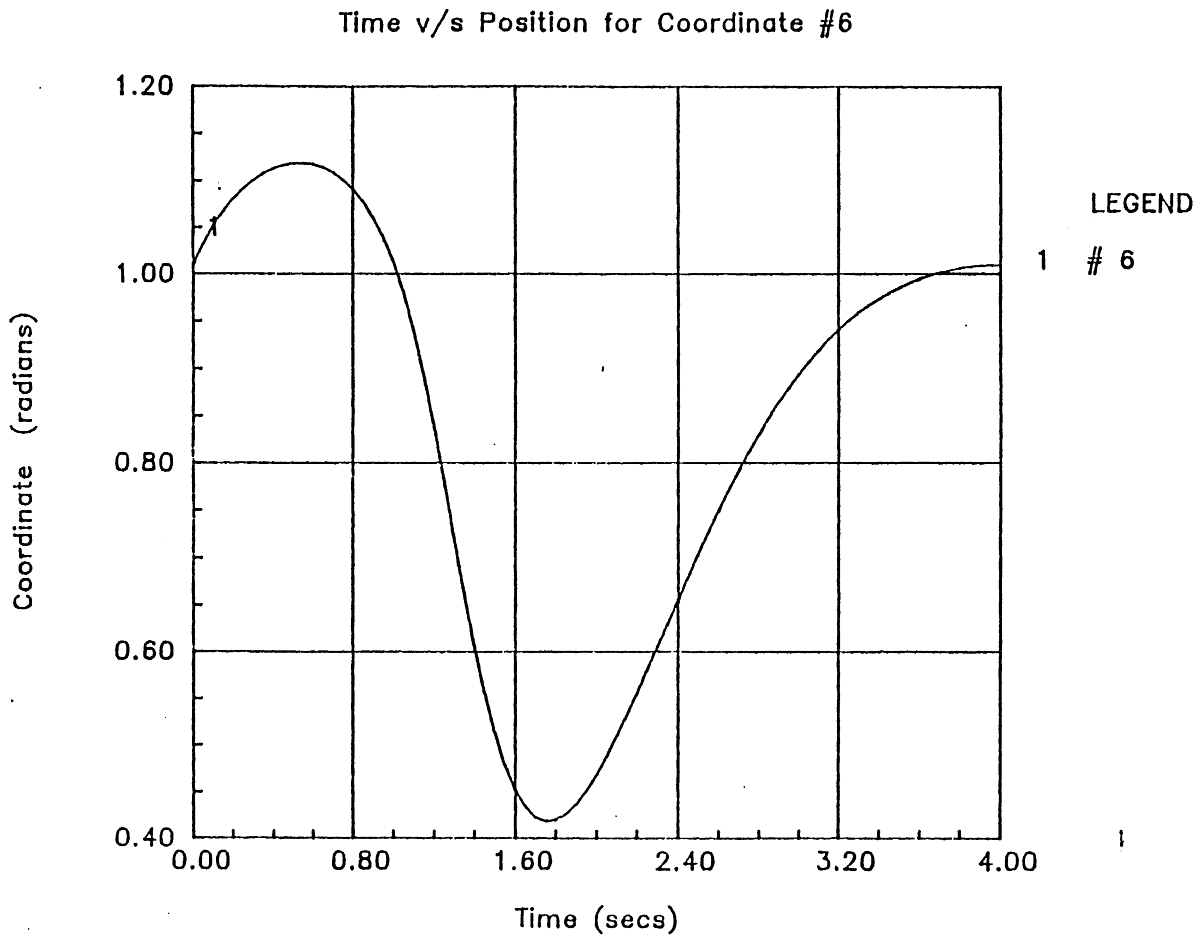

Figure 11. A Plot of Position v/s Time Coordinate #6

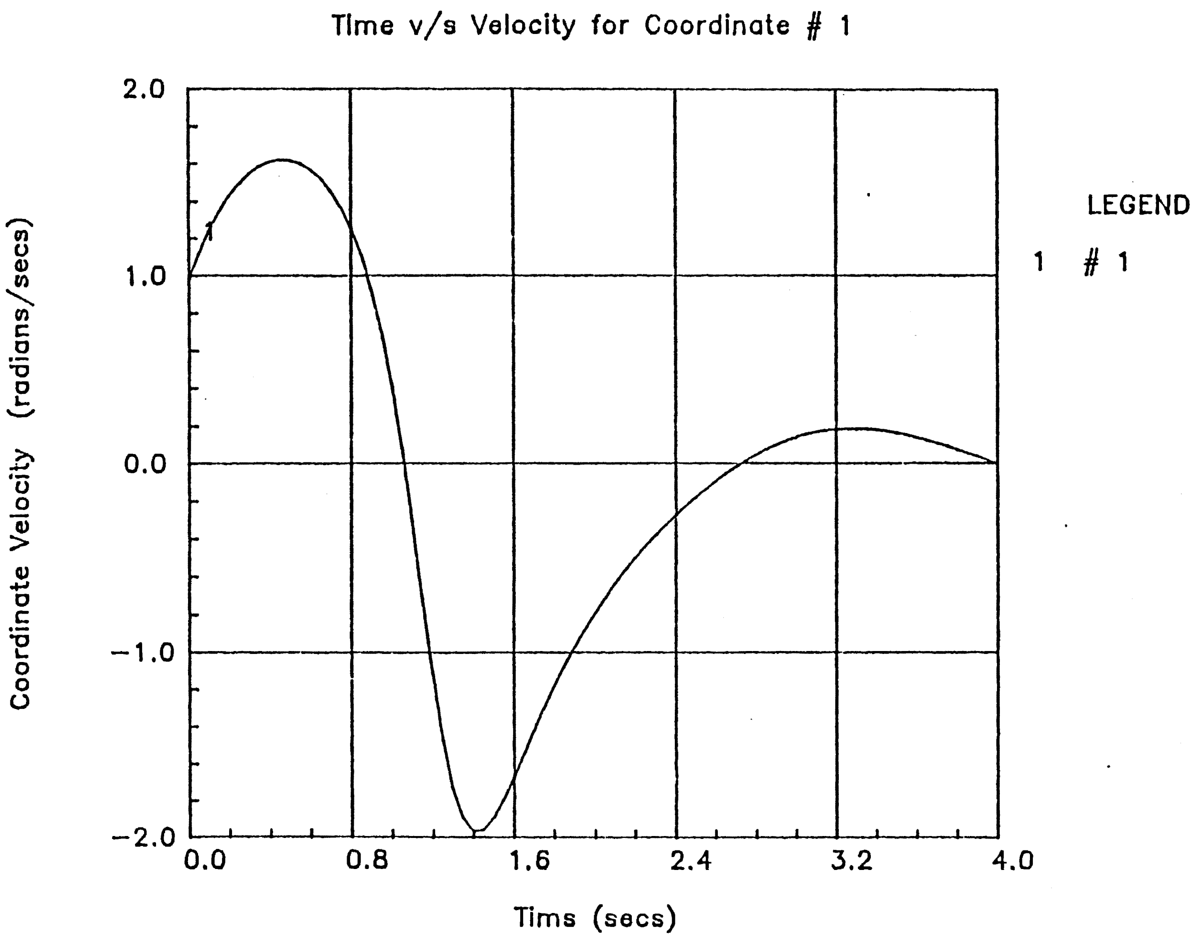

Figure 12. A Plot of Velocity v/s Time Coordinate #1

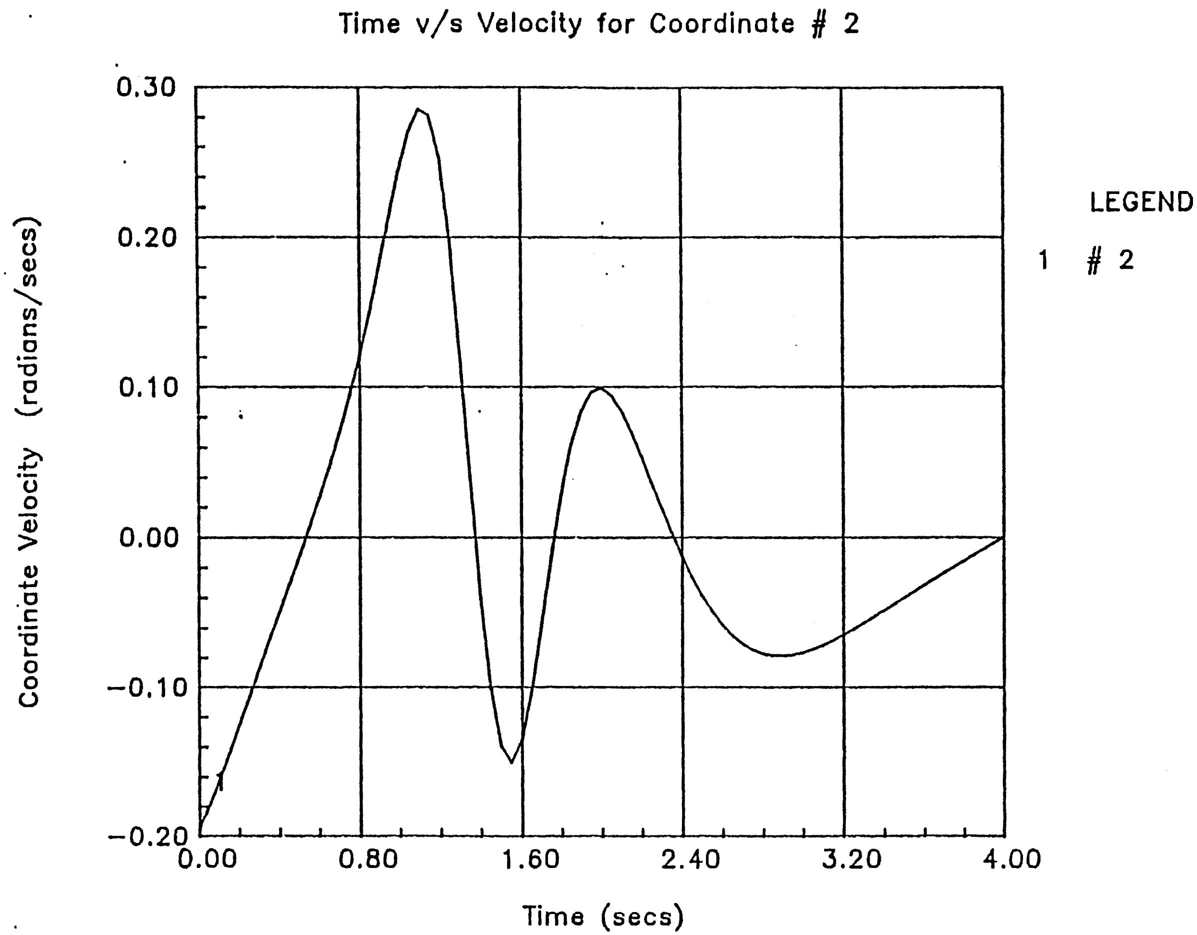

Figure 13. A Plot of Velocity v/s Time for Coordinate #2

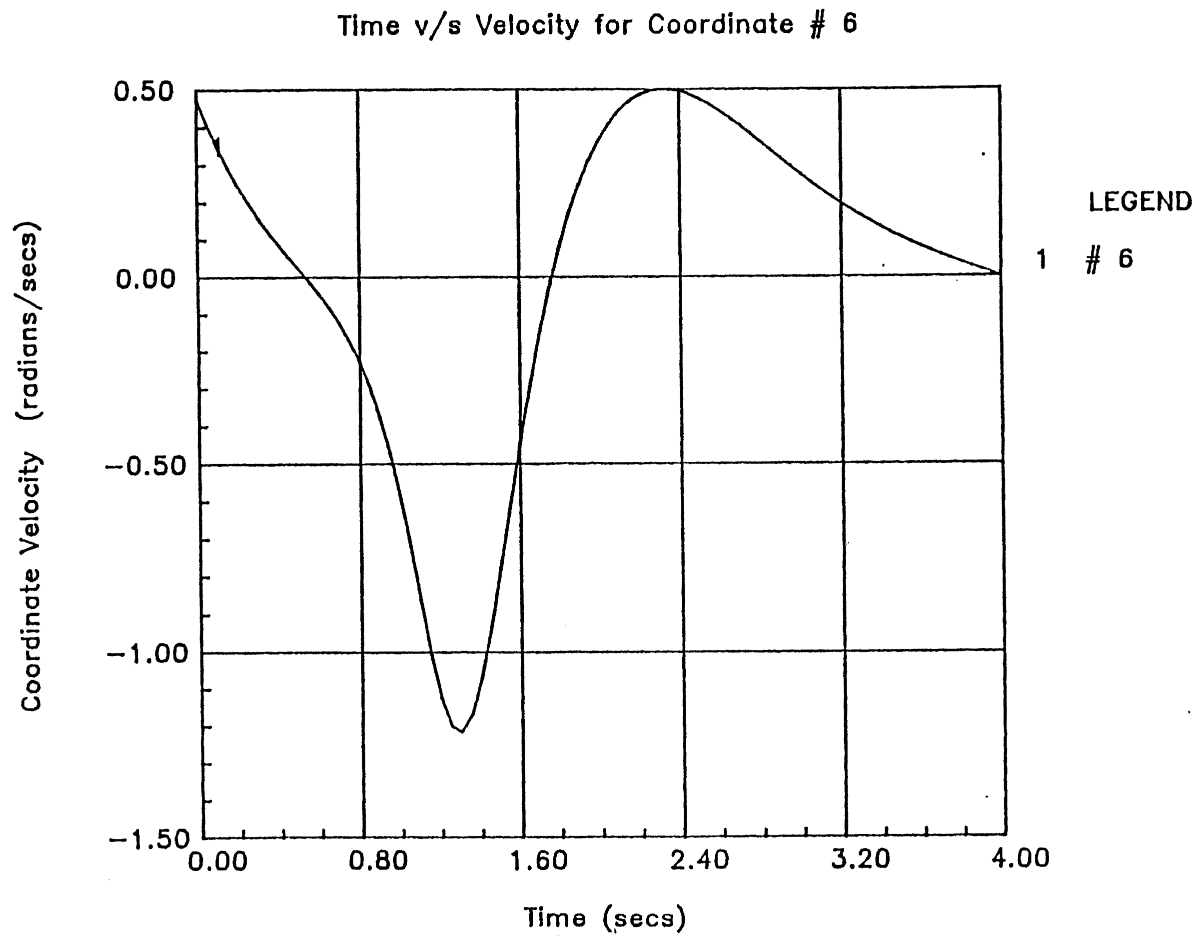

Figure 14. A Plot of Velocity v/s Time for Coordinate #6

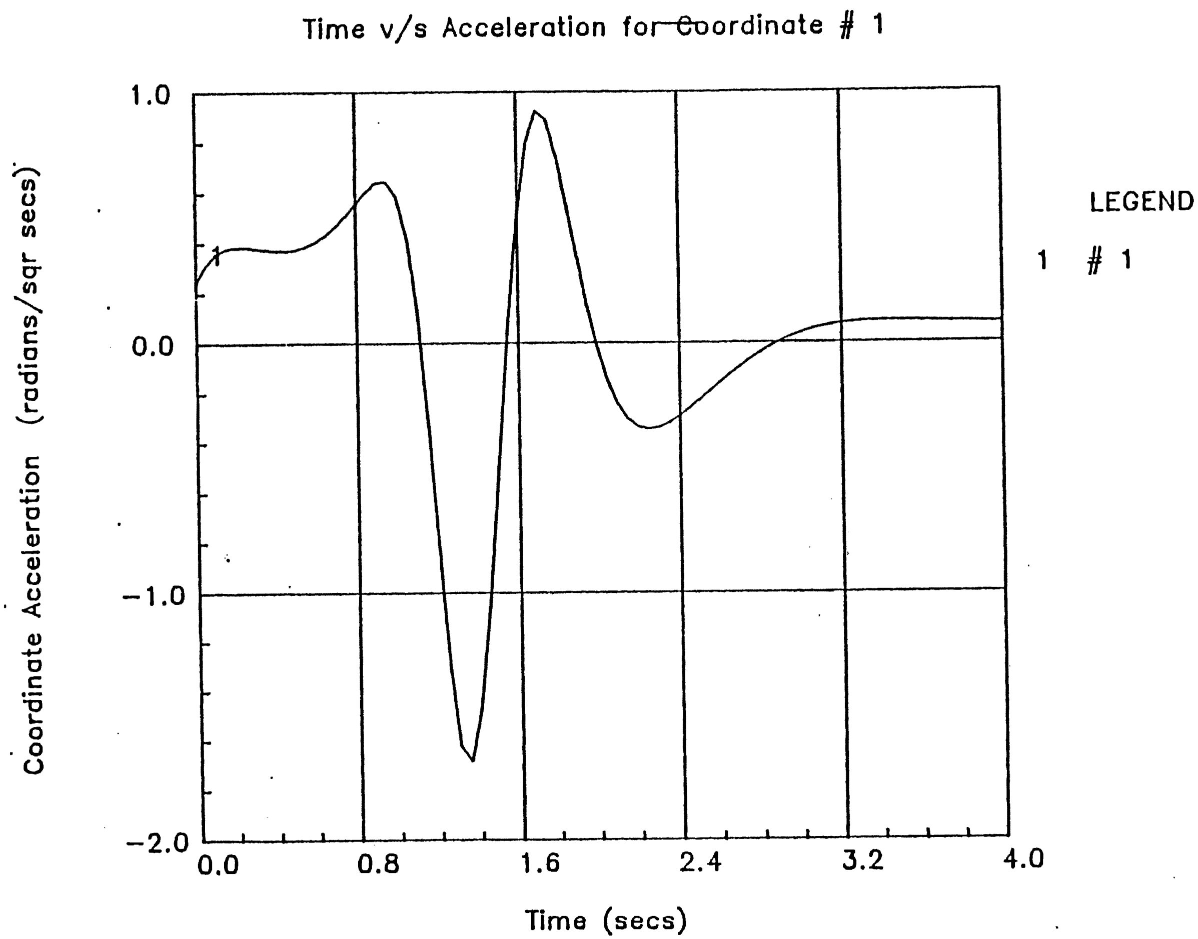

Figure 15. A Plot of Acceleration v/s Time for Coordinate #1

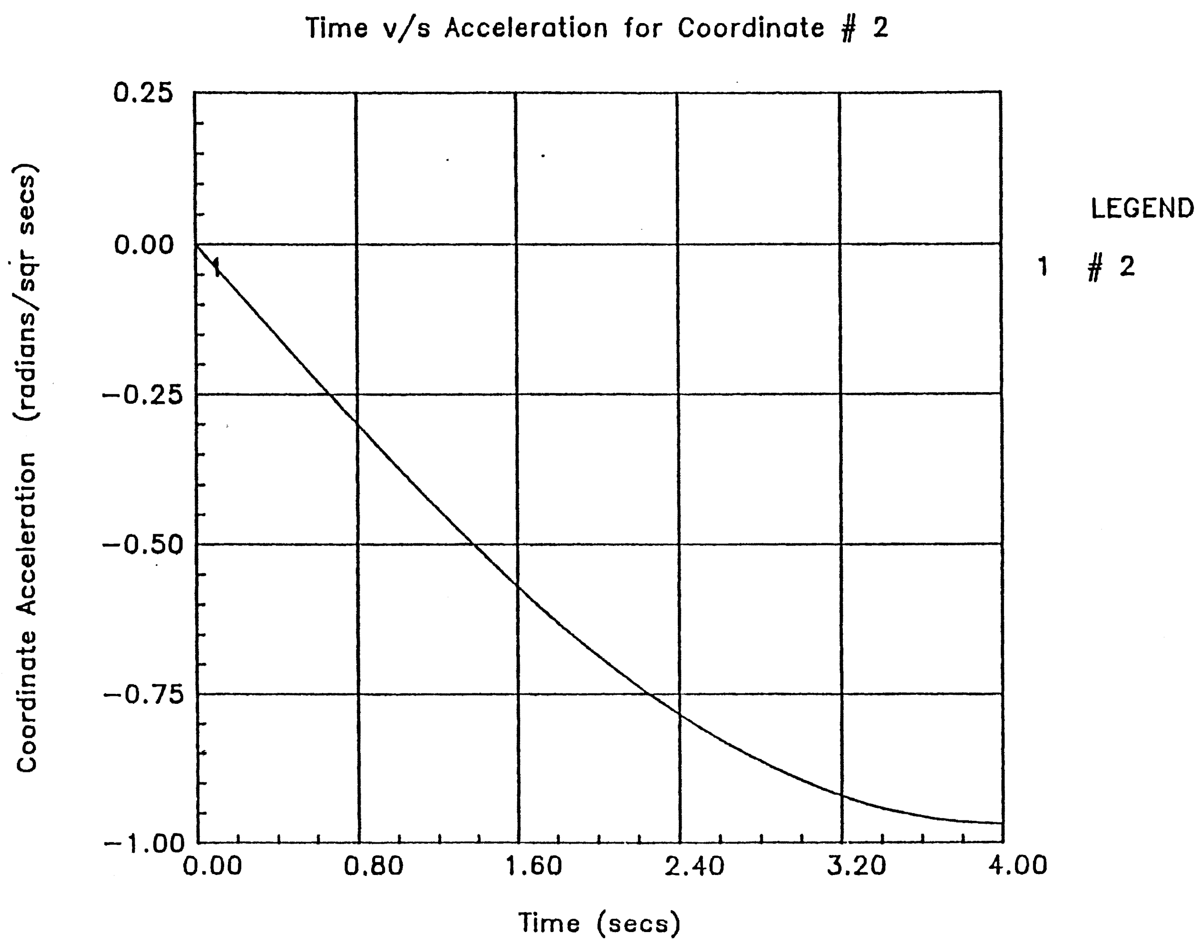

Figure 16. A Plot of Acceleration v/s Time for Coordinate #2

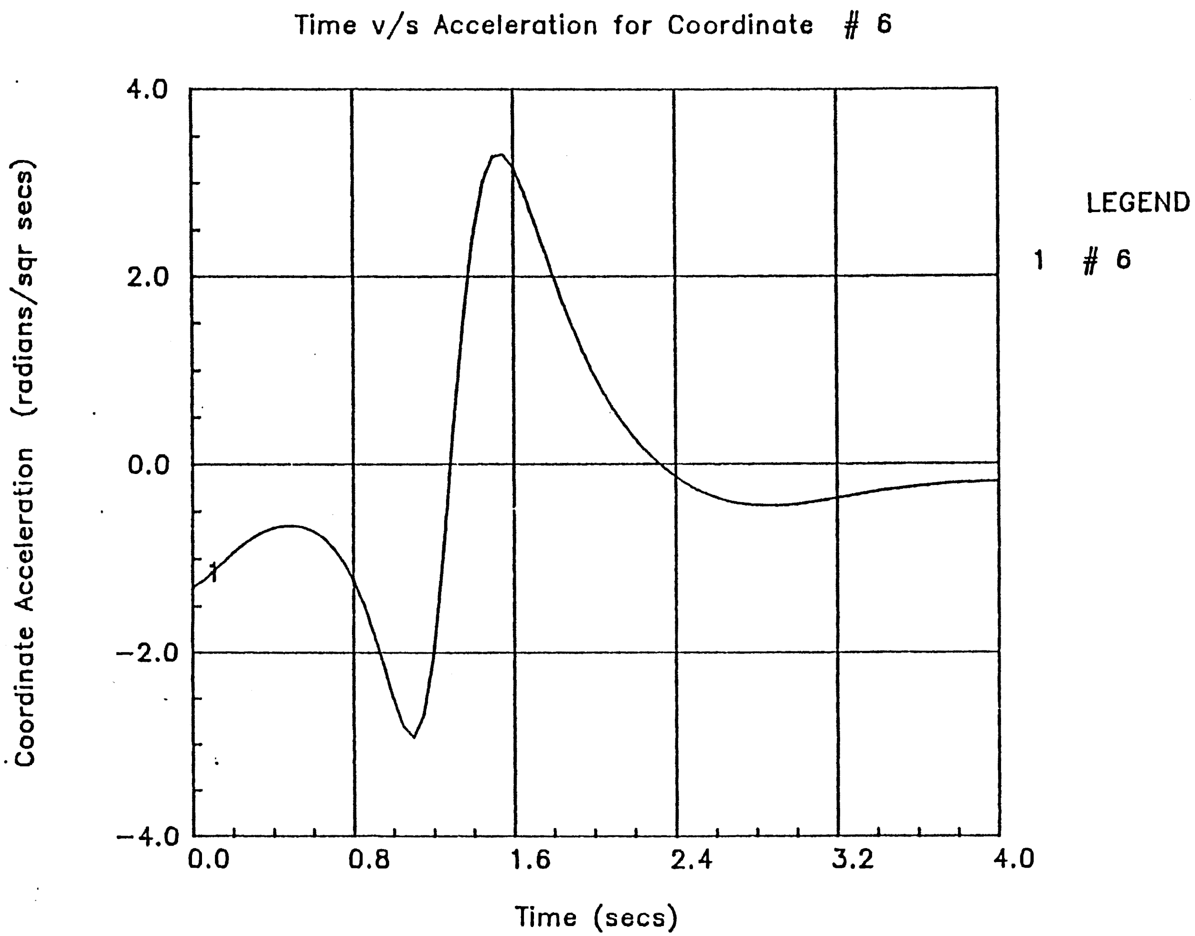

Figure 17. A Plot of Acceleration v/s Time for Coordinate #6

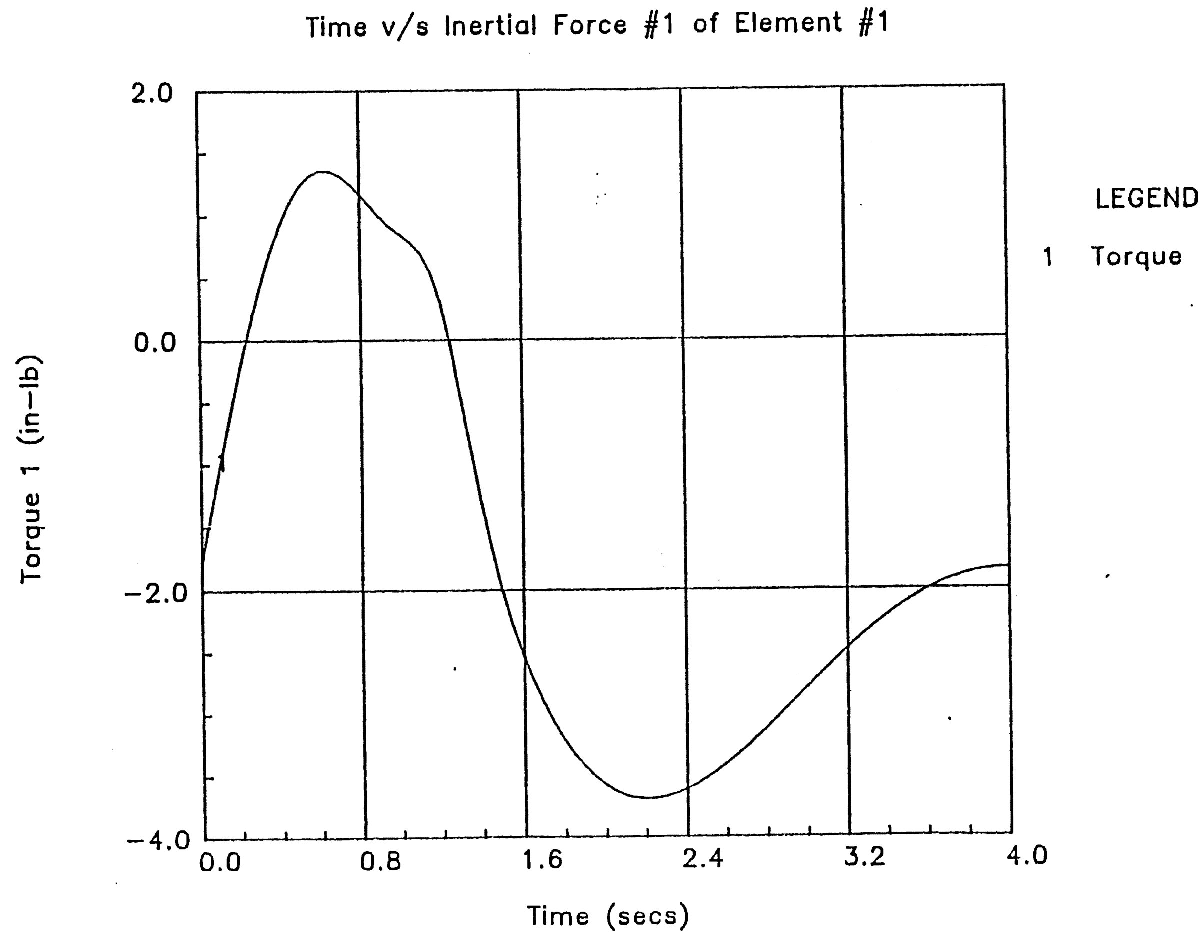

Figure 18. A Plot of Inertial Force #1 v/s Time of Element #1

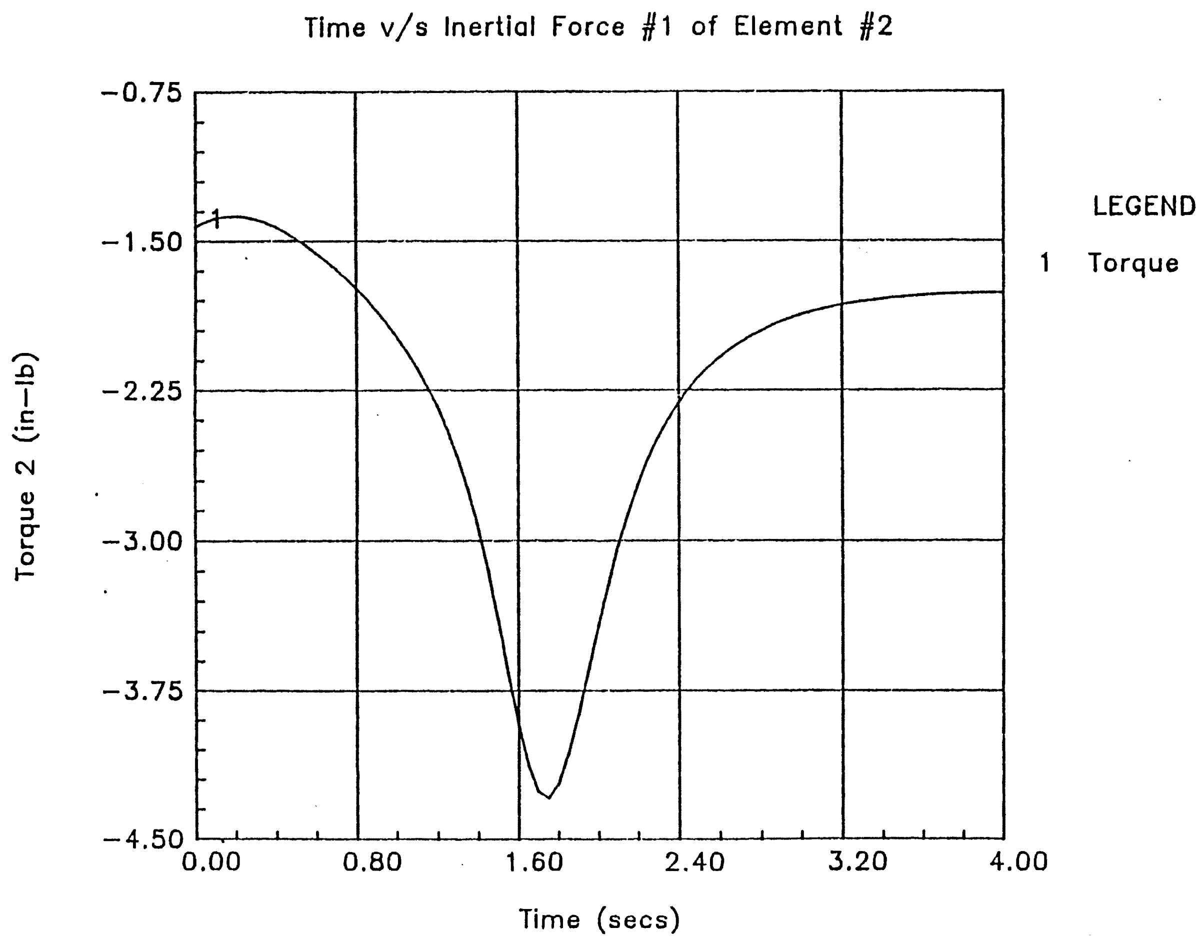

Figure 19. A Plot of Inertial Force #1 v/s Time of Element #2

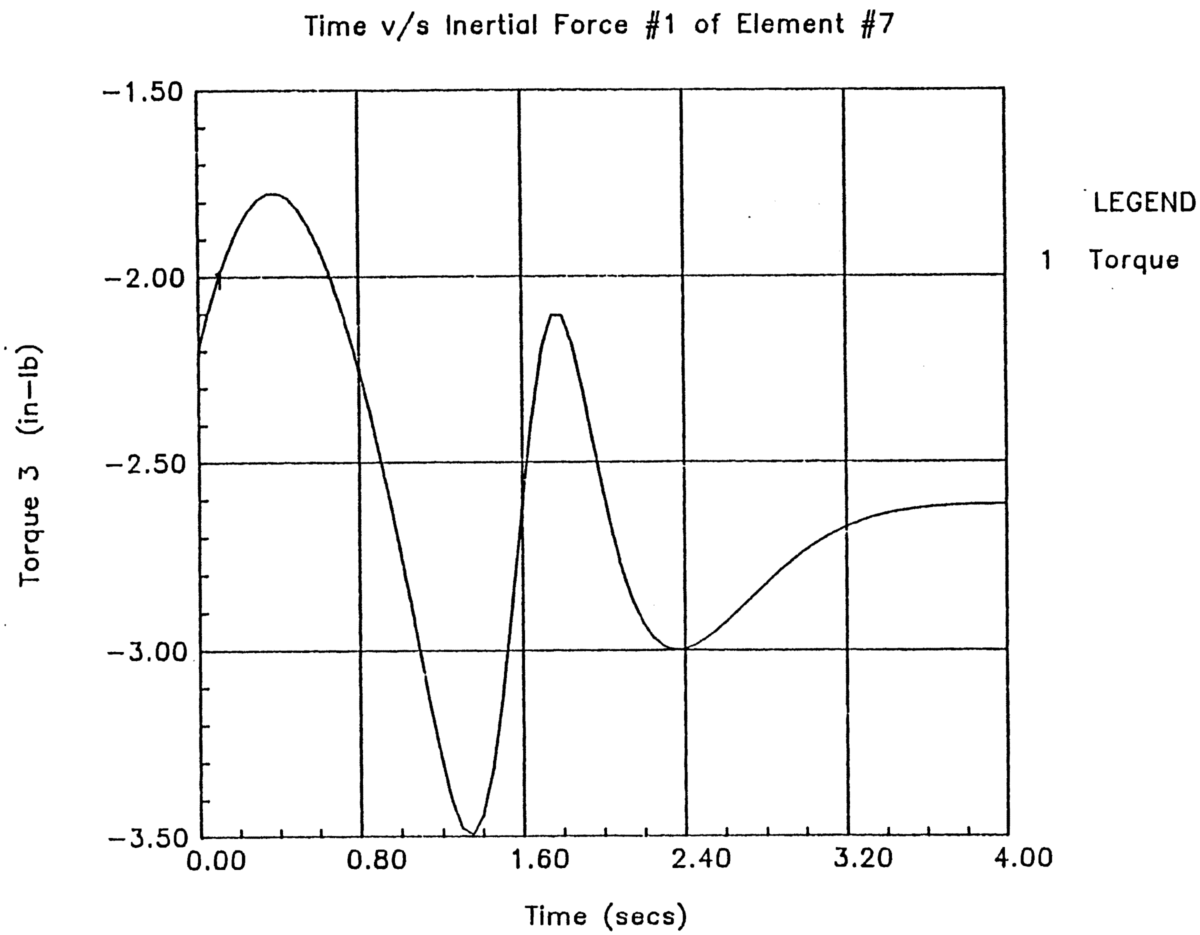

Figure 20. A Plot of Inertial Force #1 v/s Time of Element #7

$$pts(1,5) = \partial s/\partial \Phi_5 = -1$$

$$dpts(1,5) = d/dt\, \partial s/\partial \Phi_5 = 0$$

The results of the dynamic analysis are shown on figures 9 - 20. The position v/s time plots for Φ_1, Φ_2 and Φ_6 are figures 9 - 11. The velocity v/s time plots are shown on figures 12 - 14. Acceleration v/s time for the same are figures 15 - 17. The Driving torques v/s time for the three actuators are shown on figures 18 -20.

Illustrative Examples - Inverse Static Equilibrium Analysis

Example 1

This example demonstrates the static equilibrium inverse analysis, in its simplest form. It illustrates the use of the program to solve problems with one degree of freedom and one closed loop. The fig. 21 illustrates the four bar linkage and the fig. 22, the linkage in input data form or in a kinematic network form. The following are the specifications supplied to the program -

force # 1 = 50 lbs.

force # 2 = -13.29 lbs.

force # 3 = 10 lbs.

primary coordinate # 1 (value) = 0.261799 rads.

link # 1 length = 4 in.

link # 2 length = 6 in.

link # 3 length = 7 in.

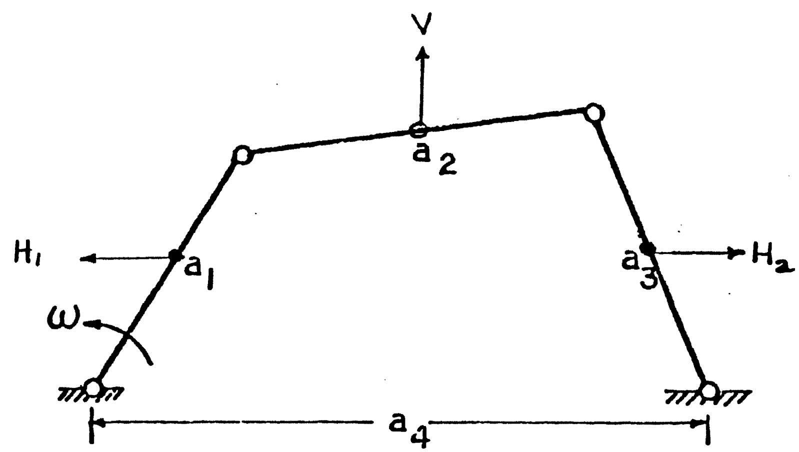

Figure 21. A Four Bar Mechanism with External Forces

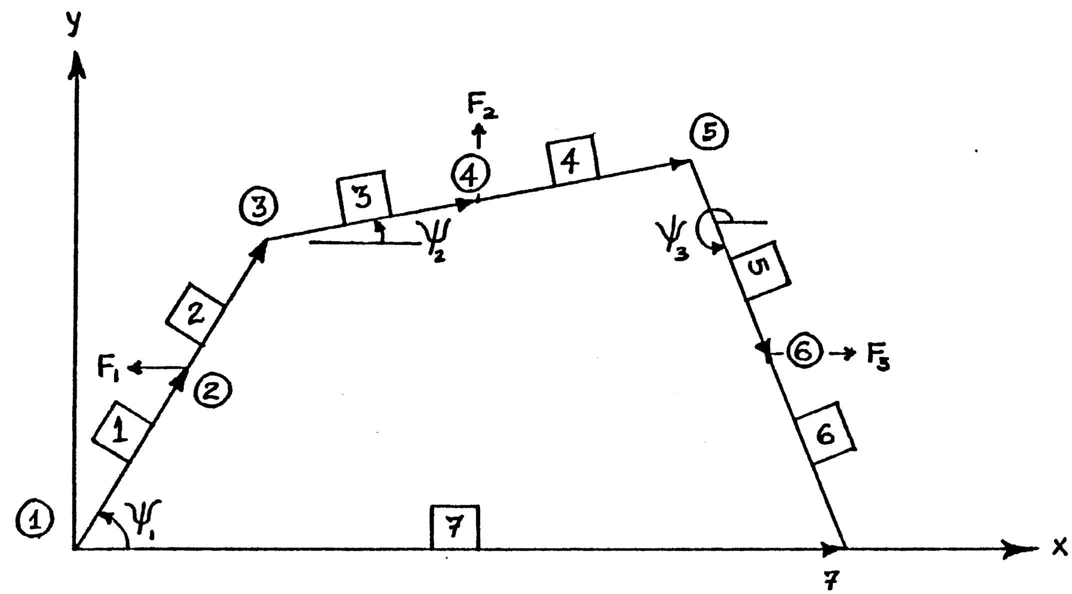

Figure 22. A Kinematic Network of a Four Bar Mechanism.

TABLE 1

COORDINATE VALUES FOR THE FOUR BAR LINKAGE EXAMPLE

No.	Coordinate No.	Coordinate Values
1	1	0.261799
2	2	5.410521
3	3	1.260848
4	4	0.000000

link # 4 length = 11.723 in.

The table 1 shows the result of the computation of this example. Note, with reference to fig. 22 that force # 1 corresponds to the vertical force at joint # 4, the force # 2 to the horizontal force at joint # 2 and force # 3 to the horizontal force at joint # 6. The primary coordinate is Φ_1. The links 1, 2, 3 and 4 are a_1, a_2, a_3 and a_4 respectively.

Example 2

The above example is a single degree of freedom problem and quite trivial solve for, using the program. This example solves a problem with two degrees of freedom as shown in the fig. 23, which illustrates the closed loop kinematic chain and the fig. 24 gives the kinematic network for the same. The problem may be briefly described as a kinematic closed loop chain, with forces 1, 2, and 3 acting at joints A, B, and C respectively. The required results are the values of Φ_1, Φ_2, Φ_3 and Φ_4.

The input data are -

force #1 = 5 lbs.

force #2 = 10 lbs.

force #3 = 5 lbs.

primary coordinate #1 (value) = 5.375614 rads.

primary coordinate #2 (value) = 4.318119 rads.

link #1 (length) = 20 in.

link #2 (length) = 20 in.

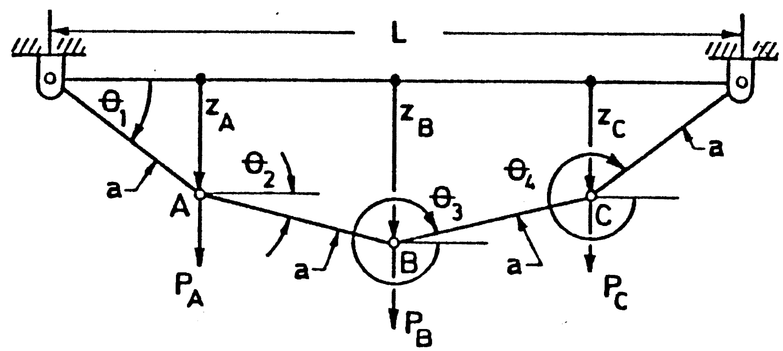

Figure 23. A Closed Loop Kinematic Chain

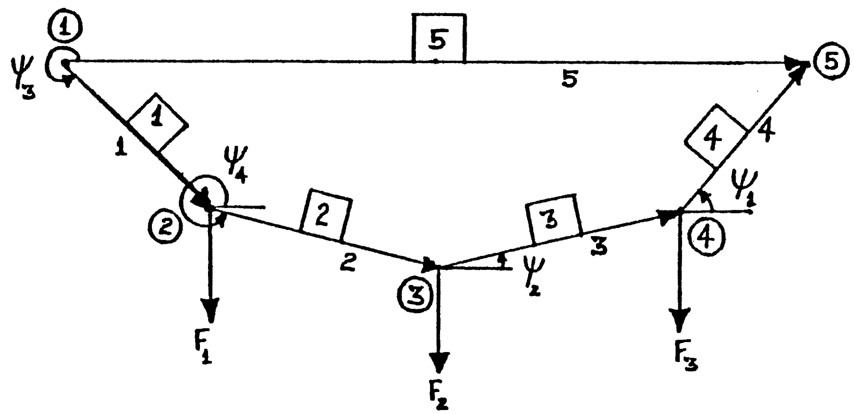

Figure 24. A Kinematic Network of the Closed Loop Chain

TABLE 2

COORDINATE VALUES FOR THE CLOSED LOOP
KINEMATIC CHAIN EXAMPLE

No.	Coordinate No.	Coordinate Values
1	1	1.965066
2	2	0.907571
3	3	5.375614
4	4	4.318119
5	5	0.000000

link #3 (length) = 20 in.

link #4 (length) = 20 in.

link #5 (length) = 40 in.

The table 2 shows the result of the example. Note unlike the problem #1 this cannot be solved explicitly. In fig. 24 the forces 1, 2, and 3 act at joint #'s 2, 3 and 4 respectively. The primary coordinates are Φ_1 and Φ_2 respectively.

Example 3

Both the above examples do not illustrate any sliders. This example shows the solution of the vertical slider crank mechanism shown in fig. 25 and its kinematic network in fig. 26. The number of degrees of freedom are two. The links are equal in length. The slider is weightless and the forces are acting at B and C and are equal. The input data is as follows -

force #1 = 4.4 lbs.

force #2 = 0.9 lbs

primary coordinate #1 = 1.0253809 rads.

primary coordinate #2 = 2.0391691 rads.

link #1 (length) = 4 in.

link #2 (length) = 4 in.

link #3 (length) = 4 in.

The Table 3 shows the computed result for this problem. With reference to fig. 26 the forces 1 and 2 act at joints 2 and 3 and the two primary coordinates are lagrangian

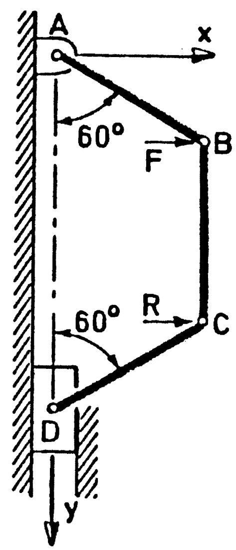

Figure 25. A Vertical Slider Crank Mechanism

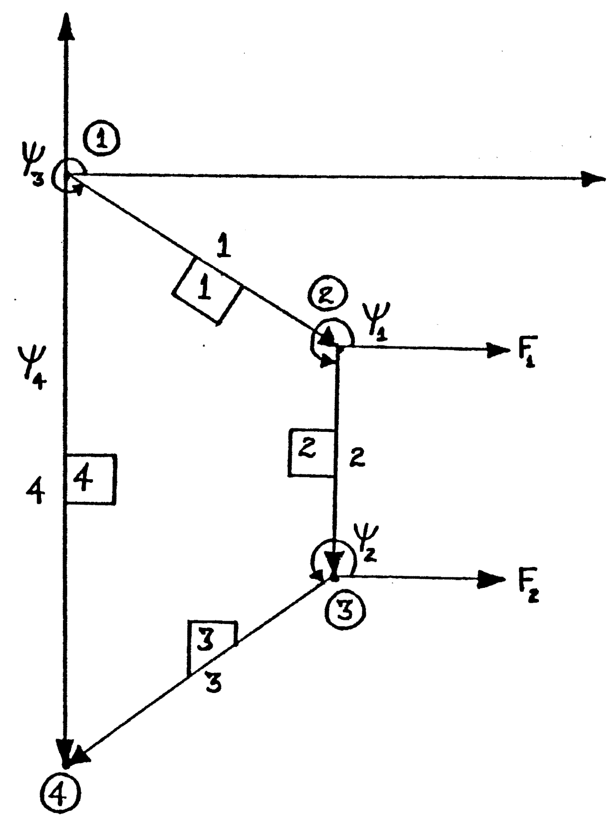

Figure 26. A Kinematic Network of the Vertical Slider Crank Mechanism

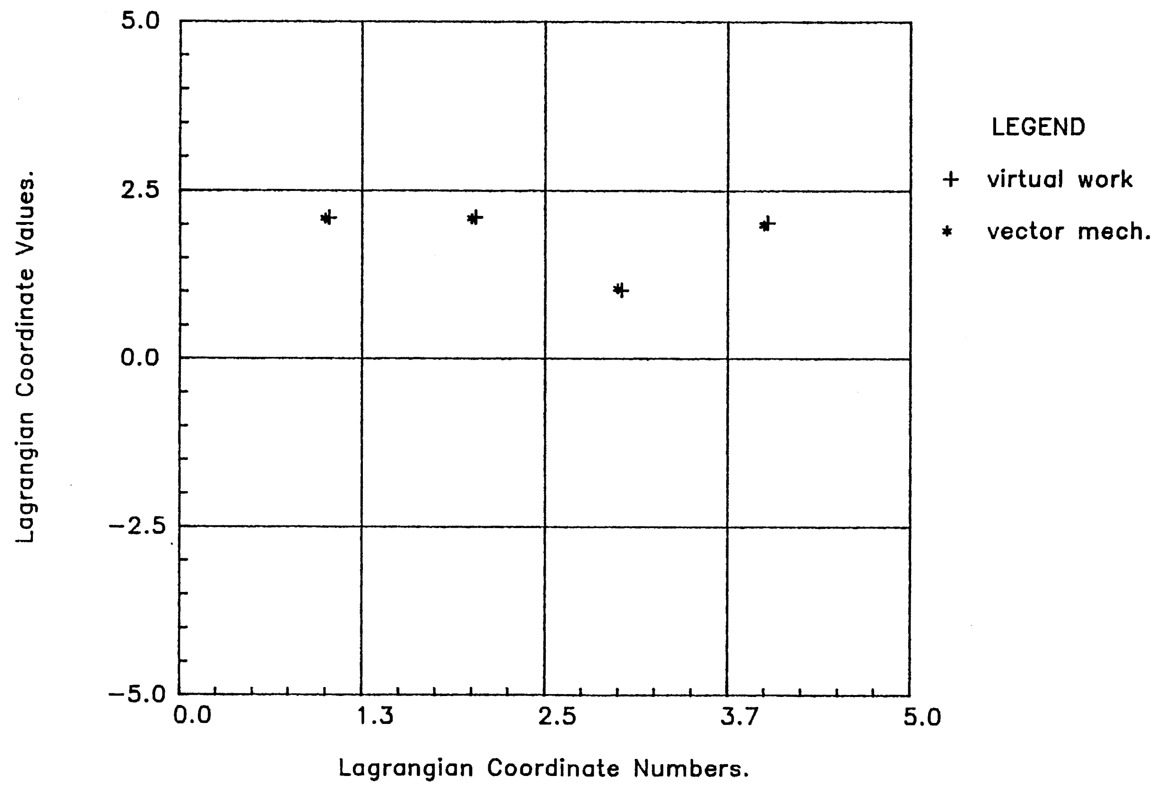

Figure 27. A Verification Plot of the Vertical Slider Crank Mechanism Example

TABLE 3

COORDINATE VALUES FOR THE VERTICAL SLIDER
CRANK MECHANISM EXAMPLE

No.	Coordinate No.	Coordinate Values
1	1	2.1057379
2	2	2.1162117
3	3	1.0253809
4	4	2.0391691

coordinates 3 and 4. The above example is verified and compared with the solution arrived at using the vector mechanics approach, Amin (1979), in fig. 27.

<u>Example 4</u>

The following example illustrates the use of this program to solve a multiloop mechanism. The fig. 27 shows the mechanism and the fig. 28 gives the kinematic network for the same. The mechanism data supplied is -

force #1 = 100 lbs.

force #2 = 79.29 lbs

primary coordinate #1 = 5.497787 rads.

primary coordinate #2 = 1.047197 rads.

link #1 (length) = 2 in.

link #2 (length) = 4 in.

link #3 (length) = 3 in.

link #4 (length) = 2 in.

link #5 (length) = 0 in. (initial estimate)

link #6 (length) = 5.6 in.

link #7 (length) = 6.4 in.

The Table 4 shows the output generated by the program for the above problem. In the fig. 28 the force 1 acts at the joint 6 and the force 2 at the joint 3. The primary coordinates 1 and 2 are lagrangian coordinates 1 and 2.

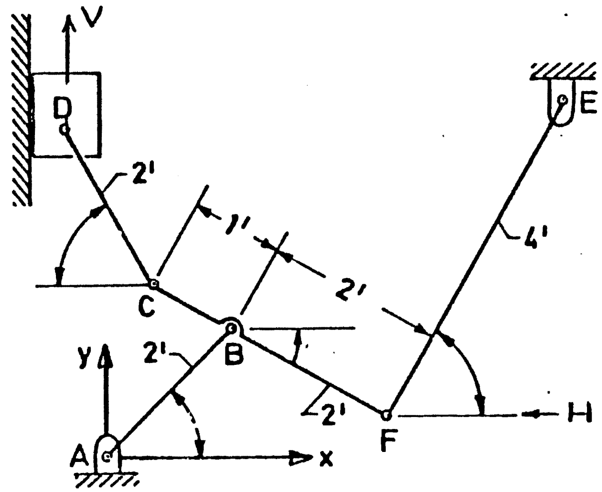

Figure 27. A Multi Loop Mechanism

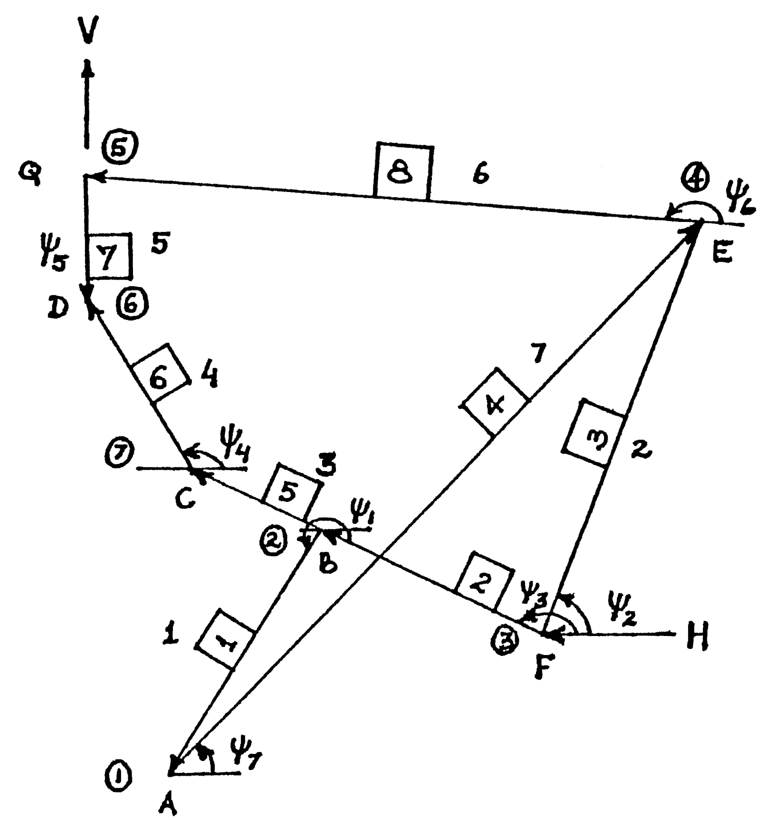

Figure 28. A Multi Loop Mechanism

TABLE 4

COORDINATE VALUES FOR THE MULTI-LOOP MECHANISM EXAMPLE

No.	Coordinate No.	Coordinate Values
1	1	5.497787
2	2	1.047197
3	3	5.759586
4	4	2.094395
5	5	0.250000
6	6	0.000000
7	7	0.610865

CHAPTER VI

SUMMARY AND CONCLUSIONS

The main objective of this work was to examine the avenues for a generalized approach for inverse static equilibrium analysis, using the virtual work principle. Enhancing the status of the existing dynamic analysis of mechanisms package was essential to incorporate the inverse static equilibrium analysis and assimilate the different techniques that have been used therein for mechanism analysis.

In applying the virtual work principle, it is assumed that the planar mechanisms are in static equilibrium and successive iterations solve for the equilibrium position, if the mechanism does not comply to the assumption made. The procedure used expresses the mechanisms's virtual displacement for all langrangian coordinates in terms of the primary and the secondary coordinates and computes the virtual work done by these virtual displacements. The Newton-Raphson technique is used to numerically iterate towards the final result. The problem mathematically gives rise to a highly nonlinear set of equations, which exhibits great resistance to convergence.

Vector mechanics approach is the usual method employed for inverse static equilibrium analysis and as with the virtual work principle, the vector mechanics approach also gives rise to a set of highly nonlinear equations. The advantage of virtual work principle is in the fact that it creates a smaller set of equations as compared to the vector mechanics approach, thus leading to a faster convergence.

This analysis has been developed for planar mechanisms. It may be extended to spatial mechanisms, however in case of spatial mechanisms, for the ease of computation it would be better to use the matrix approach rather than the complex number approach. Also the use of links that are not ideal and conditions that express friction and other external parameters are desirable. Viewing the program from a programmer's outlook, such large computation intensive programs require a fast computer, and a system capable of the essential interactive computer graphics facilities.

BIBLIOGRAPHY

Amin, A. U., "Automatic Formulation and Solution Techniques in Dynamics of Machinery." Ph.D. Dissertation, University of Pennsylvania, 1979.

Curtis, F. G. and Wheatley, P. O., "Applied Numerical Analysis." Addison Wesley Publishing Company, 1983.

Dado, M. and Soni, A. H., "Complete Dynamic Analysis of Elastic Linkages" Journal of Mechanisms, Transmissions, and Automation in Design, Trans. ASME no. 86-DET-52, 1986.

Livermore, D. F., "Analysis of Large Deflections in Multiple Degree of Freedom Suspension Systems" Ph.D. Dissertation, Northwestern University, 1965.

Orlandea, N. and Chase, M. A., "Simulation of a Vehicle Suspension with Adams Computer Program." SAE paper no. 770053, SAE Warrendale, PA, 1977.

Paul, B., "Kinematics and Dynamics of Planar Machinery." Prentice Hall, 1979.

Razi, R., "Static Force Analysis of Spatial Mechanisms by the Matrix Method." M.S. Thesis, Northwestern University, 1963.

Segerlind, L. J., "Applied Finite Element Analysis." John Wiley and Sons, 1984.

Sheth, P. N. and Uicker, J. J., " IMP (Integrated Mechanisms Program), A Computer Aided Design Analysis System for Mechanisms and Linkages." Journal of Engineering for Industry, Trans. ASME ser.B, vol. 92, pp. 454-464, 1972.

Soni, A. H., "Mechanisms Synthesis and Analysis", Scripta Book Company, McGraw-Hill Book Company, 1974.

APPENDIX A

PROGRAM LAYOUT AND STRUCTURE

APPENDIX A

PROGRAM LAYOUT AND STRUCTURE

The program has the following new features -

It has been installed on the Silicon Graphics Iris 3030, a graphics workstation, thus it has an excellent graphics capability. The optimum use of the Iris's graphics ability has been made in this project. The program is completely Interactive. The use of C programming language's vast capability has allowed the programmer to give complete interactive benefits with least programming effort. At the same time the number crunching part of the program is executed in Fortran and C, the reason being, to use the rich math and built-in functions in Fortran and the compact structure available in C. This use of C and Fortran in a single program is possible by the "Fortran to C and C to Fortran" interface provided by the Iris's compiler.

The other feature is the division of the program into files. None of the files are over a thousand lines of code, for a programmer this is a boon especially when the code is very large and the programmer wishes to access complete control over it. The splitting up of the code, to be stored in different files is possible due to the fact that the operating system of the workstation, Unix, has the "makefile" function.

The program files are as described below -

file 1 - Main file - ma1.f.

This file contains the main program for dynamic analysis.

file 2 - Preprocessor1 file - pr1.c.

This file contains the interactive input code for dynamic analysis.

file 3 - Postprocessor1 file - po1.f.

This file contains the output functions to display the completed analysis of the program.

file 4 - Graphics1 & Math1 file - gm_fun.c.

This file contains the all the graphic and math functions available for dynamic analysis.

file 5 - Functions1 file - su11.f.

This file has all the subroutines and functions necessary for the number crunching sections of the dynamic analysis.

file 6 - Main2 file - ma2.c.

This file contains the main program for inverse static equilibrium analysis, it has been configured as a C language function to be accessible by the dynamic analysis's main program and to be accessible as one package.

file 7 - Math2 file - mfun.c.

This file contains the all the math functions available for inverse static equilibrium

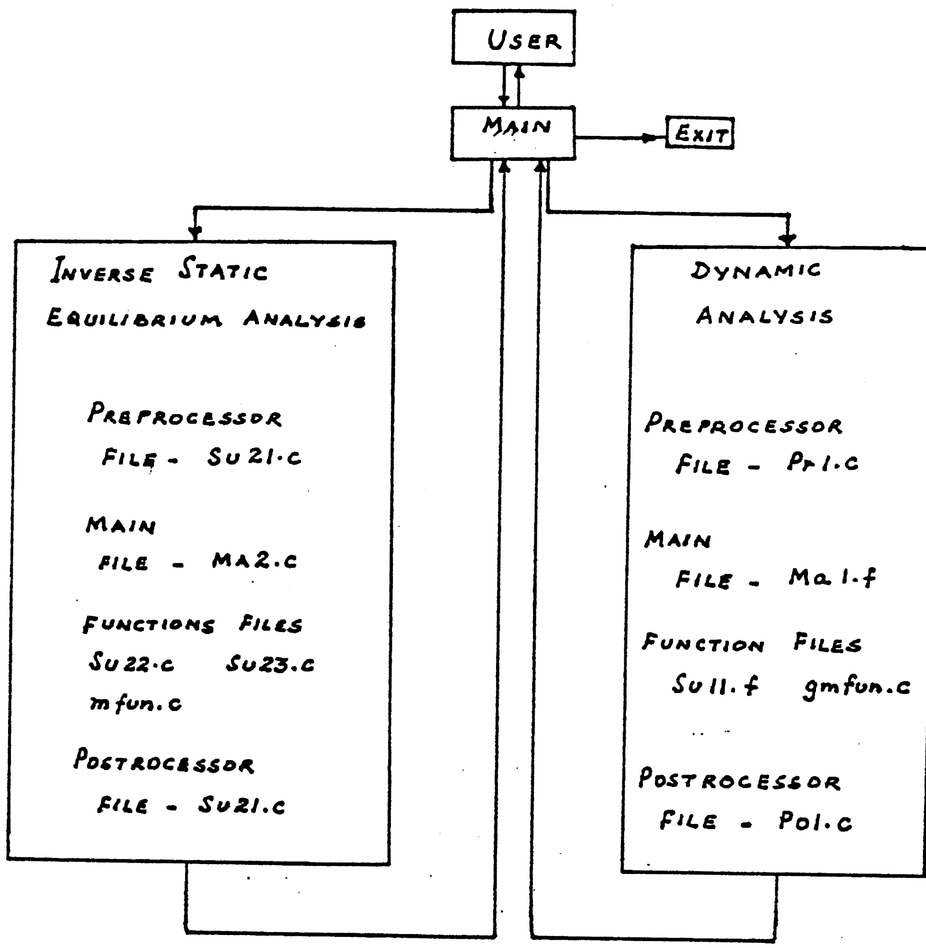

Figure 30. Flowchart of the Complete Program

analysis.

file 8 - Preprocessor2 & Postprocessor2 file - su21.c.
This file contains the preprocessor and the postprocessor for inverse static equilibrium analysis. These functions are used for interactive input and output of completed analysis and data.

file 9 - Functions file a - su22.c.
This file and the following one contain the number crunching functions for inverse static equilibrium analysis.

file 10 - Functions file b - su23.c.

The fig. 27, gives the program's flowchart this may be vital for any further improvements to be done on this program.

VITA

RAGHU DWARAKANATH

Candidate for the Degree of

Master of Science

Thesis: A GENERALIZED INVERSE STATIC EQUILIBRIUM ANALYSIS FOR PLANAR MECHANISMS

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born in New Delhi, India, September 9th 1962. The son of Mr. K. R. Dwarakanath and Mrs. P. Dwarakanath.

Education: Graduated from Saint Mary's School, Nairobi, Kenya, in January 1979; received Bachelor of Engineering degree from the Malaviya Regional Engineering College, Jaipur, Rajasthan, India in January 1985; completed requirements for Master of Science Degree from Oklahoma State University, December, 1987.